

1 **The Rise, Fall and Stabilization of US Inflation:**
2 **Shifting Regimes and Evolving Reputation***

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5 **Abstract**

6 The rise, fall, and stabilization of US inflation between 1969 and 2005
7 is consistent with a model of shifting policy regimes that features a
8 forward-looking New Keynesian Phillips curve, policymakers that can
9 or cannot commit, and private sector learning about policymaker type.
10 Using model-implied inflation forecasting rules to extract state variables
11 from the inflation forecasts in the Survey of Professional Forecasters,
12 we provide evidence that policy regimes without commitment prevailed
13 before 1980 and regimes with commitment prevailed afterward. With
14 theory and quantification, we find that evolution of reputational capital
15 is central to understanding the behavior of inflation.

16 *Keywords:* time inconsistency, reputation game, optimal monetary pol-
17 icy, forward-looking expectations

18 *JEL classifications:* E52, D82, D83.

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19 “In reality, however, the anchoring of inflation expectations has
20 been a hard-won achievement of monetary policy over the past few
21 decades, and we should not take this stability for granted. [...] a
22 policy of achieving “temporarily” higher inflation over the medium
23 term would run the risk of altering inflation expectations beyond
24 the horizon that is desirable. Were that to happen, the costs of
25 bringing expectations back to their current anchored state might
26 be quite high.”

27 (Donald L. Kohn, “Monetary Policy Research and the Financial
28 Crisis: Strengths and Shortcomings”, October 9, 2009)

29 1 Introduction

30 The interplay of inflation, policy and expectations is at the heart of modern
31 macroeconomics. Figure 1 plots one-quarter-ahead inflation forecast errors
32 based on the Survey of Professional Forecasters from 1969 to 2005. Around
33 1980, the average inflation forecast error turns from persistently positive to
34 persistently negative, a feature which is highlighted by an 8 quarter moving
35 average of the errors (the black dashed line).¹ Many authors have used this
36 pattern of runs of forecast errors as evidence for private sector learning with
37 a mis-specified model of the economy.²

38 We instead view the pattern as arising from private sector’s gradually learn-
39 ing about the type of policymaker in place.³ Before 1980, we see a U.S.
40 policymaker that could not commit and thus continually produced positive
41 inflation surprises. After 1980, we see a policymaker committed to carrying
42 out promised inflation plans, but that faced private sector skepticism so that
43 inflation was frequently below forecasts, particularly in the early years.

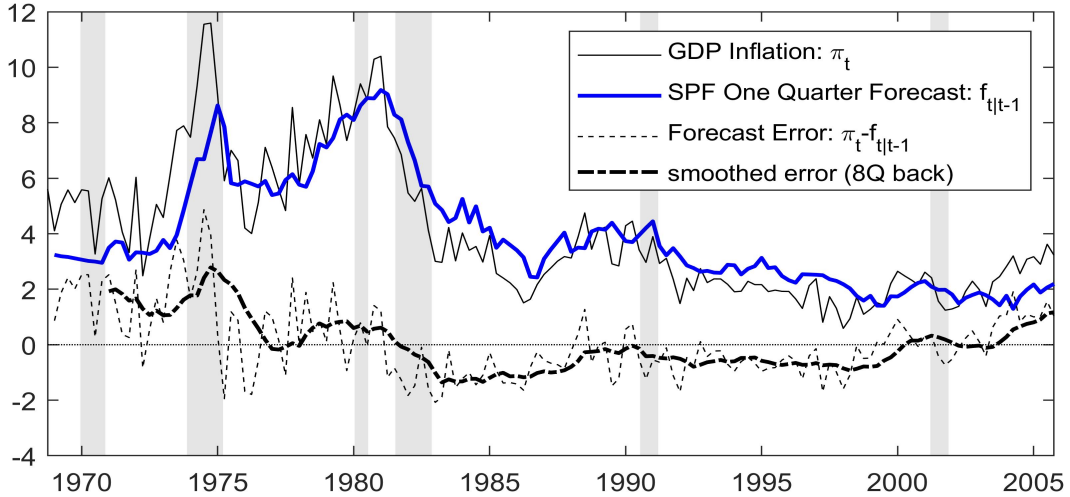
44 Building a model to turn these simple ideas into macroeconomic time se-
45 ries is the main research activity reported in this paper. Our model has four

¹We thank Donghai Zhang for pointing us to this observation.

²See G.W. Evans and Honkapohja (2008), Woodford (2013) and Eusepi and Preston (2018) for the surveys of this literature. Relative to our work, this approach captures different aspects of uncertainty regarding economic fundamentals.

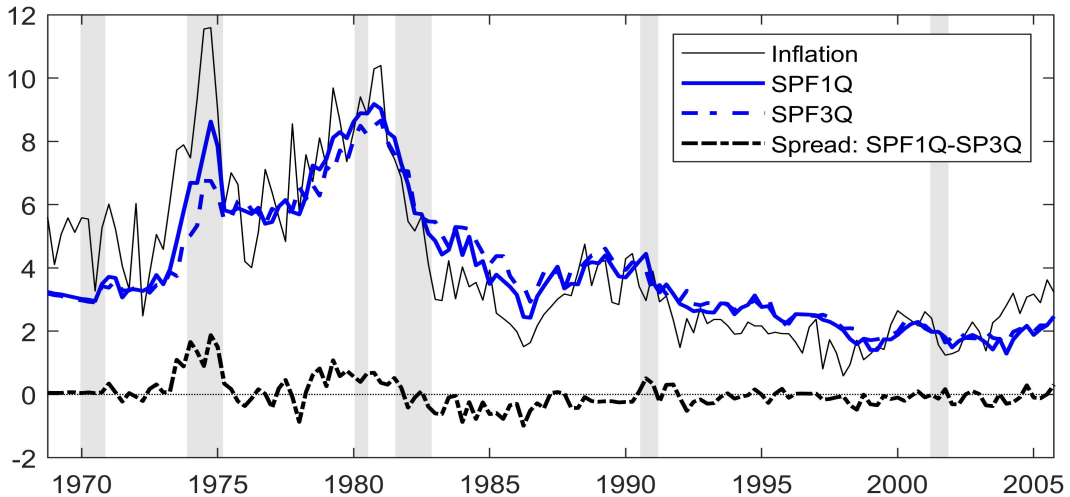
³Matthes (2015) develops a model in which the private sector learns to differentiate between two competing monetary policymaking approaches, while our setup highlights how purposeful policy depends on private sector learning.

Figure 1: Forecast error of inflation



The GDP inflation rate, π_t , and the Survey of Professional Forecasters median inflation forecast rate made one quarter earlier, $f_{t|t-1}$, rise and fall together over 1968Q4 through 2005Q4. Inflation is notoriously difficult to forecast so that the forecasting error, $\pi_t - f_{t|t-1}$, is volatile, although it averages close to zero over the sample period. The errors display serial correlation – lengthy runs of positive and negative forecasting values – that are highlighted by an 8 quarter moving average.

Figure 2: SPF and SPF Spread



The one quarter ahead forecast $f_{t+1|t}$ and the three quarter ahead forecast $f_{t+3|t}$ rise and fall together. The SPF spread $f_{t+3|t} - f_{t+1|t}$, which will play an important role in the paper, also displays sustained intervals of high or low values. All variables are continuously compounded annualized rates of change. Additional detail on macroeconomic data and the SPF constructions are provided in Appendix C.

46 main structural elements: a forward-looking New Keynesian Phillips curve,
47 policymakers that can or cannot commit, Bayesian learning by the private
48 sector about policymaker type, and occasional observable changes in regime.⁴
49 Combining our model-implied inflation forecasting rules with the term struc-
50 ture of inflation forecasts in the Survey of Professional Forecasters, we provide
51 evidence that policy regimes without commitment prevailed before 1980 and
52 regimes with commitment prevailed afterward. In our theory and quantifica-
53 tion, evolution of reputational capital is central to the behavior of expectations
54 and inflation.

55 In establishing this understanding of inflation history, we make three con-
56 tributions. First, we develop a recursive model of optimal intended inflation
57 by a policymaker capable of commitment, facing private sector suspicion that
58 there may be an alternative, optimizing but myopic, policymaker in place.⁵
59 This committed policymaker’s reputation - the private sector’s likelihood that
60 there is a committed type in place - is a key model state variable that evolves
61 in accordance with optimal Bayesian learning. While we’ve previously used
62 recursive methods to investigate optimal policy with and without reputation
63 dynamics, the essential new wrinkle is that the alternative policymaker is not
64 a machine, but is purposeful (opportunistic).⁶ Formally, this adds an incentive
65 compatibility constraint to our recursive approach: it turns out to be one with
66 substantial content for reputation evolution and inflation.

67 Second, we show that model-implied inflation forecasts at various horizons
68 are functions of state variables, including the highly persistent reputation state
69 and a more temporary price shock. We then design an empirical strategy to
70 extract these states utilizing SPF forecasts at multiple horizons. Figure 2 high-
71 lights the smoother nature of the three-quarter-ahead SPF forecast of inflation
72 relative to the one-quarter-ahead forecast.⁷ Taking a cue from literature on
73 the term structure of interest rates, we form an SPF spread, plotted as the
74 black dashed line and defined as the difference between the one and three

⁴Coibion et al. (2018) advocate developing inflation models using survey data like the SPF, focusing on the New Keynesian Phillips curve that confronts our policymaker.

⁵“Intended inflation” captures our policymaker’s imperfect control of inflation: see below.

⁶Our label matches Mishkin (2008) and other work on commitment and communication.

⁷Elmar Mertens guided us to the SPF term structure via Mertens and Nason (2020).

75 quarter forecasts.⁸ The interest rate term structure analogy suggests that this
76 spread should be positive when there are persistent, but ultimately tempo-
77 rary increases in inflation.⁹ Our empirical design extracts unobserved states
78 from the SPF term structure, exploiting the information used by sophisticated
79 forecasters at the time.

80 Third, as the intended inflation choices of committed and opportunistic
81 policymakers are functions of these state variables, we can trace out the his-
82 tory of these intentions over 1969-2005 without taking a stand on the type of
83 policymaker in place at any date. However, in our theory, observed inflation
84 should differ from intended inflation by a serially uncorrelated shock. Looking
85 at the early history through the end of the 1970s, we find an average deviation
86 of about zero from the opportunistic type's intended inflation but a strongly
87 positive average deviation from the committed type's intended inflation: we
88 thus conclude that the early interval was comprised of regimes without com-
89 mitment. Looking after 1982, we find the reverse pattern: the deviations
90 average about zero for the committed type and are negative on average for the
91 opportunistic type: we thus conclude that the latter interval was comprised of
92 regimes with commitment.

93 A roadmap to the paper is as follows. Section 2 locates our work within
94 the literature. Section 3 describes our model economy, while section 4 defines
95 its perfect Bayesian equilibrium. Section 5 highlights the decision problem of
96 the committed type, which is more complicated than in many optimal policy
97 setups due to incentive constraints and reputation dynamics, but shows it fits
98 neatly into a recursive equilibrium. Section 6 begins with model calibration
99 and extraction of latent states. It then shows that our model-implied policy
100 measures capture US inflation's rise, fall, and stabilization between 1970 and
101 2005.¹⁰ Section 7 develops the interaction of reputation and policy. Section 8
102 constructs measures of policy credibility and derives a counterfactual inflation
103 history assuming a single committed regime. Section 9 concludes.

⁸Under the expectations theory, the comparable spread would be the one quarter ahead forward rate less the three quarter ahead forward rate.

⁹Conceptually, this description is consistent with a stationary autoregressive component. Empirically, the SPF spread rises in Figure 2 during the 1974-75 inflation surge.

¹⁰Primiceri (2006) provides an alternative account that focuses on policymaker, rather than private agent, learning.

2 Literatures

Our work has links to history, macroeconomic theory and econometrics.

2.1 Excessive and rising inflation without commitment

The stagflation of the mid 1970s led to an explosion of new theoretical ideas about macroeconomic policy. Some leading macroeconomic theorists argued that excessive inflation was rooted in the interaction of private agent inflation expectations with a policymaker that couldn't commit to future actions (Kydlan and Prescott (1977)). Stagflation itself would arise if inflation bias intensified with a higher natural rate of unemployment (Barro and D.B. Gordon (1983a)). Sargent (1982) argued that regime change was important for stopping sustained inflation, large and small. Yet, while these ideas attained great prominence, many economists have expressed doubt about the historical importance of commitment capacity, reputation evolution, and regime shifts.¹¹

Our model provides a positive theory of “The Great Inflation” that was the objective of the early literature and relies on its basic insights, while stressing private sector learning. It does not rely on a time-varying natural rate of unemployment and involves only a small amount of intrinsic inflation bias (a policymaker's choice with well-anchored expected inflation). Its core mechanism for increasing inflation is the positive feedback between expectations and the choices of a policymaker that can't commit: under full information rational expectations, the feedback leads to a bias of 8% or more at the Nash equilibrium in the terminology of Sargent (1999) and others.¹²

But in our model, the rise in inflation can be very gradual as learning slows down the positive feedback mechanism. More specifically, if a policymaker that cannot commit begins with a good reputation, rising actual inflation can have relatively minor effects on expected inflation because the private sector

¹¹For example, Blinder (1997) is skeptical about basic inflation bias mechanism. While Parkin (1993) found some support for the Barro-Gordon hypothesis, Levin and Taylor (2013) argued against it by depicting timing misalignment between changes in actual and expected inflation and changes in the natural rate of unemployment. Ireland (2004) found low frequency co-movement between inflation and unemployment consistent with the Barro-Gordon setup, but his econometric analysis didn't support its more detailed time series implications.

¹²That is, if it is evident that a new policymaker is unable to commit, there would be an immediate jump to a high inflation rate until another regime change.

130 believes that the committed type is most likely in place and expects future
131 anti-inflation policies. Further, faced with inflation expectations that are low
132 and stable, the opportunistic type opts not to increase inflation much because
133 its intrinsic bias is small. In turn, this makes for slow learning and a gradual
134 drift in the level of actual and expected inflation as in Figure 1. Yet, our
135 model also predicts that temporary positive price shocks – such as those at
136 several points in the 1970s – can speed up the erosion of reputation that must
137 ultimately occur when there isn’t commitment.

138 Being myopic, our opportunistic type does not have reputation concerns,
139 but private agents never the less may face difficulty in distinguishing it from
140 a committed policymaker.¹³

141 2.2 Commitment, disinflation and inflation stabilization

142 Another strand of literature investigates optimal choices by a policymaker that
143 is endowed with the ability to commit but faces a skeptical private sector.
144 Cukierman and Liviatan (1991) and R.G. King et al. (2005) employ 1980s-
145 type models,¹⁴ and focus on whether a dramatic “cold turkey” or smoother
146 “gradualist” disinflation strategy is desirable for a committed type managing
147 expectations in the face of private sector skepticism and evolving reputation.¹⁵

148 Our model features the forward-looking New Keynesian Phillips curve, pri-

¹³Reputational dynamics were also an important element in the 1980s literature, but for a reason very different from our analysis. Barro and D.B. Gordon (1983b), Backus and Driffill (1985b) and Backus and Driffill (1985a), and Barro (1986), showed that reputational forces can substitute for commitment capability, leading a “discretionary” policymaker to behave like a committed one that mechanically adopts a low inflation rule. This early work led to a major literature on “sustainable plans” for patient policymakers in environments without commitment (Chari and Kehoe (1990)). Recently, Dovis and Kirpalani (2021) extends this literature to a situation where there is uncertainty about whether the policymaker can commit ex post. Insights from these studies will be important to our ongoing work to introduce a long-horizon opportunistic type into our framework.

¹⁴That is, similar policy objectives and the same Lucas-style Philips curve as used by Kydland and Prescott (1977) and Barro and D.B. Gordon (1983a).

¹⁵Another dimension of these studies is on signalling equilibria, including the appropriate private sector interpretation of monetary policy announcements when these signals may be sent either by a truth-telling committed type or a dissembling alternative type. The key conclusion – reinforced by the careful work of Lu (2013) on a related fiscal model – was that a signalling equilibrium involves the truth-telling committed type announcing a policy that solves a natural optimal policy problem and the opportunistic type sending the same message. We therefore abstract from the analysis of signalling equilibria.

149 vate sector learning with imperfect public monitoring, and prospective regime
150 change.¹⁶ We build on our prior work, [Lu et al. \(2016\)](#), that studies opti-
151 mal reputation building by a committed policymaker with a non-committed
152 policymaker behaving mechanically. In the current setup, both types of policy-
153 makers are purposeful. The optimal response of a non-committed policymaker
154 significantly changes the reputation building incentives of the committed pol-
155 icymaker: a new committed policymaker with a good initial reputation will
156 follow an apparently gradualist plan, while one with a poor initial reputation
157 will select more dramatic actions.

158 **2.3 Recursive contracts and optimal policy design**

159 A methodological contribution of this paper is the development of a recursive
160 approach that determines optimal intended inflation of both types of pol-
161 icymakers, when the private sector forms forward-looking rational inflation
162 expectations. We conceive of the committed policymaker as a principal who
163 chooses state-contingent plans for his own actions and those of the two agents
164 – the private sector and the opportunistic policymaker – subject to a rational
165 expectation constraint for the former and an incentive compatibility constraint
166 for the latter. We follow the recursive contracts literature to recast the op-
167 timization problem in a recursive form,¹⁷ but modify the standard approach
168 by employing a “change of measure” to tackle a particular challenge in our
169 setup where one agent, the private sector, disagrees with the principal – the
170 committed policymaker – about the probabilities of specific future histories.

171 **2.4 Time series econometrics**

172 There are interesting connections of our work to prominent studies of inflation
173 and inflation forecasting using reduced form and structural models.

174 *Stochastic trends:* Many econometric models of inflation contain a stochas-
175 tic trend.¹⁸ The estimated stochastic trends are sometimes interpreted as
176 time-varying inflation targets, either structurally or informally.

¹⁶[Bianchi \(2012\)](#), [Debortoli and Nunes \(2014\)](#), [Debortoli and Lakdawala \(2016\)](#) also de-
velop models where agents anticipate a possible policy regime change.

¹⁷[Khan et al. \(2003\)](#), [Golosov et al. \(2016\)](#) and [Marcet and Marimon \(2019\)](#).

¹⁸A few examples are [Erceg and Levin \(2003\)](#), [Smets and Wouters \(2003\)](#), [Ireland \(2004\)](#),
[Stock and Watson \(2007\)](#), and [Cogley et al. \(2010\)](#).

177 Our model has only two structural shocks, both stationary, and there is
178 a constant target inflation rate that serves as a long-run objective for the
179 committed type. Yet, within a regime, reputation evolves as a martingale
180 relative to the information set of private agents which includes both actual
181 inflation and price shocks. Reputation is one of our model’s state variables –
182 influencing both intended and actual inflation in a nonlinear manner – so that
183 it can potentially impart an apparent stochastic trend to inflation.

184 *Markov switching:* A policymaker in our model is one of two types. We
185 assume that this type is not observed by private agents, although the dates
186 of regime switches are public information. Our setup is thus similar to the
187 “Markov switching” models pioneered by [Hamilton \(1989\)](#), but with some
188 important differences. In a standard Markov switching model, agents learn
189 about a hidden state when incoming data is more likely to have come from
190 one state relative to another, so that learning proceeds more rapidly when
191 states are more dispersed. That makes it too easy to learn with the wide
192 variation in observed inflation rates.¹⁹

193 Crucially, in our setup, the policy difference between the two regimes varies
194 endogenously with reputation, i.e., the private sector’s belief about policy-
195 maker type. In particular, the optimal policy difference is small when the
196 private sector believes that the committed type most likely is in place, and
197 is larger when it thinks otherwise. The private sector’s belief, in turn, is de-
198 termined by past policy differences via Bayesian learning. The interplay of
199 optimal policy difference and evolving private sector beliefs allows us to make
200 learning relevant while matching the large inflation swings during the period
201 of the Great Inflation and the Volcker Disinflation.

202 **3 The Economy**

203 A policymaker designs and announces a plan for current and future inflation.
204 A private sector composed of atomistic forward-looking agents is uncertain
205 whether the policymaker can commit or not, and their forward-looking deci-

¹⁹Combining Markov switching with a stochastic trend, [M. Evans and Wachtel \(1993\)](#) develop a two-regime model to explain U.S. inflation that is immune from this feature, as one regime is a persistent but stationary process while the other is a random walk. They highlight that private agents’ learning would lead to runs of forecast errors.

206 sions reflect the possibility that an announced policy plan may not be executed.

207 **3.1 Private sector**

208 Private agents' behavior is captured by a standard NK Phillips curve

$$209 \quad (1) \quad \pi_t = \underbrace{\beta E_t \pi_{t+1}}_{e_t} + \kappa x_t + \varsigma_t,$$

210 where β is their time discount factor, $E_t \pi_{t+1}$ is their expectation about the
 211 next-period inflation (with e_t being short-hand for discounted expected infla-
 212 tion), and ς_t is a cost-push shock governed by an exogenous Markov chain with
 213 the transition probabilities $\varphi(\varsigma_{t+1}; \varsigma_t)$.

214 **3.2 Policymaker**

215 The policymaker is responsible for the inflation rate, π , but cannot control it
 216 exactly.²⁰ There are two types of policymaker. A *committed* type ($\tau = 1$)
 217 chooses and announces an optimal state-contingent plan for intended inflation
 218 at all dates when he first takes office and executes it in all subsequent periods
 219 until replaced.²¹ An *opportunistic* type ($\tau = 2$) makes the same announce-
 220 ments, but chooses his own intended inflation on a period-by-period basis.

221 The private sector does not observe the policymaker's type or his intended
 222 inflation, denoted by a_t for the committed type or α_t for the opportunistic
 223 type. Yet, it observes an inflation rate π that deviates randomly from the
 224 policymaker's intention, with a density $g(\pi_t|a_t)$ or $g(\pi_t|\alpha_t)$. We assume that
 225 these densities imply zero mean implementation errors that are i.i.d. and

²⁰We use “policymaker” rather than “central banker” to recognize that inflation policy may be the result of various actors. For example, [DeLong \(1996\)](#), [Levin and Taylor \(2013\)](#), and [Meltzer \(2014\)](#) stress various political influences on monetary policy outcomes.

²¹We specify intended inflation rather than intended output for analytical convenience. If policy instead controlled intended real aggregate demand $\underline{x}_{\tau t}$ and $x_{\tau t} = \underline{x}_{\tau t} + \sigma_{x\tau} \varepsilon_t$, the Phillips curve $\pi_t = \kappa x_t + e_t + \varsigma_t$ implies that a choice of $\underline{x}_{\tau t} = \frac{1}{\kappa}[a_t - e_t - \varsigma_t]$ leads to identical intended inflation, although certain text expressions – particularly those for inflation expectations – are more cumbersome. As in some other related studies (see, e.g., [Faust and Svensson \(2001\)](#) and [Sargent \(1999\)](#)), we abstract from policy instruments. By contrast, [Orphanides and Williams \(2005\)](#), [Cogley et al. \(2015\)](#), and [Melosi \(2016\)](#) study macroeconomic outcomes with private agent learning under an interest rate rule.

226 independent of the intended inflation: ²²

227 (2)
$$\varepsilon_{1t} = \pi_t - a_t \text{ and } \varepsilon_{2t} = \pi_t - \alpha_t.$$

228 The policymaker of type τ has the following momentary objective

229 (3)
$$u(\pi, x, \tau) = -\frac{1}{2}[(\pi - \pi^*)^2 + \vartheta_x(x - x_\tau^*)^2]$$

230 which depends on inflation π and output gap x . There is a long-run inflation
231 target π^* and a type-specific output target x_τ^* .²³ The committed type has a
232 time discount factor β_1 ; the opportunistic type is myopic.

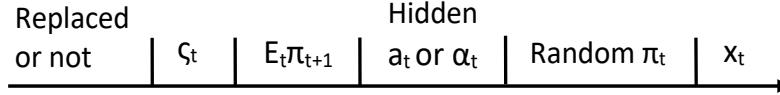
233 3.3 Timing of events

234 The within-period timing is shown in Figure 3. Private agents start period t
235 with an assessment of the probability that the incumbent policymaker is the
236 committed type, which we denote by ρ_t and call *reputation*. Next, current
237 policymaker is replaced via a publicly observed event that occurs with proba-
238 bility q , in which case the regime clock t is set to zero and the new policymaker
239 partially inherits his predecessor's reputation: $0 < \rho_0 = l(\rho_t) < \rho_t$. Then, the
240 exogenous cost-push shock ζ_t is realized. If there is a new policymaker, he
241 announces a new inflation plan. Otherwise, either type of continuing policy-
242 maker simply reiterates that current economic conditions call for an intended
243 inflation a_t . Next private agents form their expectations about the next-period
244 inflation, e_t . Then the policymaker implements the intended inflation, a_t or
245 α_t , depending on his type, which leads to a random inflation rate π_t with a
246 density $g(\pi_t|a_t)$ or $g(\pi_t|\alpha_t)$, and an output gap x_t determined by the Phillips

²²We interpret random inflation error as a reduced-form representation for all unforeseeable factors that affect the inflation rate beyond the monetary policy, following Cukierman and Meltzer (1986), Faust and Svensson (2001), Atkeson and Kehoe (2006), etc. There is also ample evidence that realized inflation rates miss the intended inflation target, with examples including Roger and Stone (2005) and Mishkin and Schmidt-Hebbel (2007).

²³The non-zero inflation target is common in central bank objectives. The output component in the objective can be written as $-\frac{\vartheta_x}{2}[x^2 + (x_\tau^*)^2] + (\vartheta_x x_\tau^*)x$ highlighting that there is a benefit to an additional unit of output. It is this composite coefficient $(\vartheta_x x_\tau^*)$ rather than its components that are important below. Our approach can easily handle publicly observable shocks to the targets π^* and x_τ^* . But since these are not essential to our analysis and have been extensively explored elsewhere, we opt for simplicity in specification.

Figure 3: Timing of events within a period



247 curve. This new information leads private agents to updated their beliefs
 248 about policymaker type, as described further below.

249 4 Macro Equilibrium in a Dynamic Game

250 Our economy consists of a private sector and a policymaker that can be one
 251 of the two types, but whose actions do not directly reveal his type: a dynamic
 252 game with incomplete information. We now define equilibrium in this game.

253 4.1 Public Equilibria

254 Define the public history of the current regime $h_t = \{h_{t-1}, \pi_{t-1}, \zeta_t\}$ as the
 255 collection of all past realizations of inflation rates and exogenous states. We
 256 restrict our attention to equilibria in which all strategies depend only on the
 257 public history, i.e., “public strategies.”²⁴ We denote the committed and oppor-
 258 tunistic policymaker’s equilibrium strategies as $a(h_t)$ and $\alpha(h_t)$, respectively.

259 4.2 Perfect Bayesian Equilibria

260 We further require the equilibrium of this incomplete information game to be
 261 perfect Bayesian. That is, the beliefs of the private sector are consistent and
 262 the strategies of the two types of policymakers satisfy sequential rationality.

263 4.2.1 Consistent beliefs: Reputation Dynamics

264 Consistency requires that the private sector’s belief about policymaker type
 265 should be updated according to the equilibrium strategies of intended inflation

²⁴Such a restriction is innocuous in our equilibrium analysis because: 1) the private sector’s strategy has to be public since h_t is its information set; 2) the committed type’s policy has to be public since it follows the announced policy plan, which needs to be verifiable by the private sector; 3) given all the other player’s strategies are public, it is also optimal for the opportunistic type to choose public strategies (Mailath and Samuelson (2006))

266 of both types of policymaker, $a(h_t)$ and $\alpha(h_t)$, the observed inflation π_t , and
 267 the Bayes' rule (4). Thus, starting from $\rho(h_0) = \rho_0$, the private sector's belief
 268 ρ is updated recursively,

$$269 \quad (4) \quad \rho(h_{t+1}) = \rho(h_t, \pi_t) \equiv \frac{\rho(h_t) g(\pi_t | a(h_t))}{\rho(h_t) g(\pi_t | a(h_t)) + (1 - \rho(h_t)) g(\pi_t | \alpha(h_t))}.$$

270 The denominator is the private sector's likelihood of a particular inflation rate,
 271 so within-regime reputation is a martingale relative to public history,

$$272 \quad (5) \quad E(\rho(h_{t+1}) | h_t) = \int \rho(h_{t+1}) [\rho_t g(\pi | a_t) + (1 - \rho_t) g(\pi | \alpha_t)] d\pi = \rho_t.$$

273 4.2.2 Consistent beliefs: Inflation Expectations

274 Consistency further requires the private sector's expectation about the next-
 275 period inflation $e_t = \beta E_t(\pi_{t+1})$ to be rational and satisfy:

$$276 \quad (6) \quad e(h_t) = \beta \left\{ \begin{array}{l} \rho(h_t) E[(1 - q) a(h_{t+1}) + qz(h_{t+1}) | h_t, \tau_t = 1] + \\ (1 - \rho(h_t)) E[(1 - q) \alpha(h_{t+1}) + qz(h_{t+1}) | h_t, \tau_t = 2] \end{array} \right\}$$

277 This expression is complicated due to the possible future regime change, which
 278 occurs with probability q . In the event of a regime change, we use z_{t+1} to
 279 denote the private sector's nowcast of inflation in period $t + 1$, and z_{t+1} in
 280 equilibrium should be

$$281 \quad (7) \quad z(h_{t+1}) = \rho_0 a(h_0) + (1 - \rho_0) \alpha(h_0) \text{ where } \rho_0 = l(\rho_{t+1})$$

282 due to the inheritance mechanism for reputation discussed above.

283 In the event of a regime continuation, the next-period inflation will de-
 284 pend on the type of current policymaker. With probability ρ_t , the current
 285 policymaker is committed, who will generate stochastic inflation π_t with den-
 286 sity $g(\pi_t | a(h_t))$ and will continue to implement the inflation plan $a(h_{t+1})$ next
 287 period. Since $h_{t+1} = \{h_t, \pi_t, \varsigma_{t+1}\}$, the conditional expectation in the first
 288 line of (6) is $E[\cdot | h_t, \tau_t = 1] = \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) (\cdot) g(\pi_t | a(h_t)) d\pi_t$. Similarly,
 289 conditional on the current policymaker being opportunistic, he will generate

290 stochastic inflation π_t with density $g(\pi_t|\alpha(h_t))$ and will implement $\alpha(h_{t+1})$
 291 next period. Hence, $E[\cdot|h_t, \tau_t = 2] = \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t)(\cdot) g(\pi_t|\alpha(h_t)) d\pi_t$.

292 4.2.3 Sequential rationality of the opportunistic type

293 An opportunistic policymaker takes private sector expected inflation $e(h_t)$
 294 as given and chooses intended inflation α each period to maximize the ex-
 295 pected objective $\int u(\pi, x, \tau = 2) g(\pi|\alpha) d\pi$, subject to the NK Phillips curve
 296 (1). Defining ι as *intrinsic inflation bias* since $\alpha = \pi^* + \iota$ if expected inflation
 297 is at target, we write the linear best response function as

$$298 \quad (8) \quad \alpha(e, \varsigma) = \pi^* + \iota + A[e - \beta\pi^*] + A\varsigma = Ae + B(\varsigma)$$

299 with $A = \frac{\vartheta_x}{\kappa^2 + \vartheta_x} < 1$, $\iota = A[\kappa x_2^* - (1 - \beta)\pi^*]$, and $B(\varsigma) = (1 - A\beta)\pi^* + \iota + A\varsigma$.

300 The top panel of Figure 4 shows the full information *Nash equilibrium*
 301 *inflation bias*, at the intersection of best response (red solid) and the 45 degree
 302 (black dash) lines,

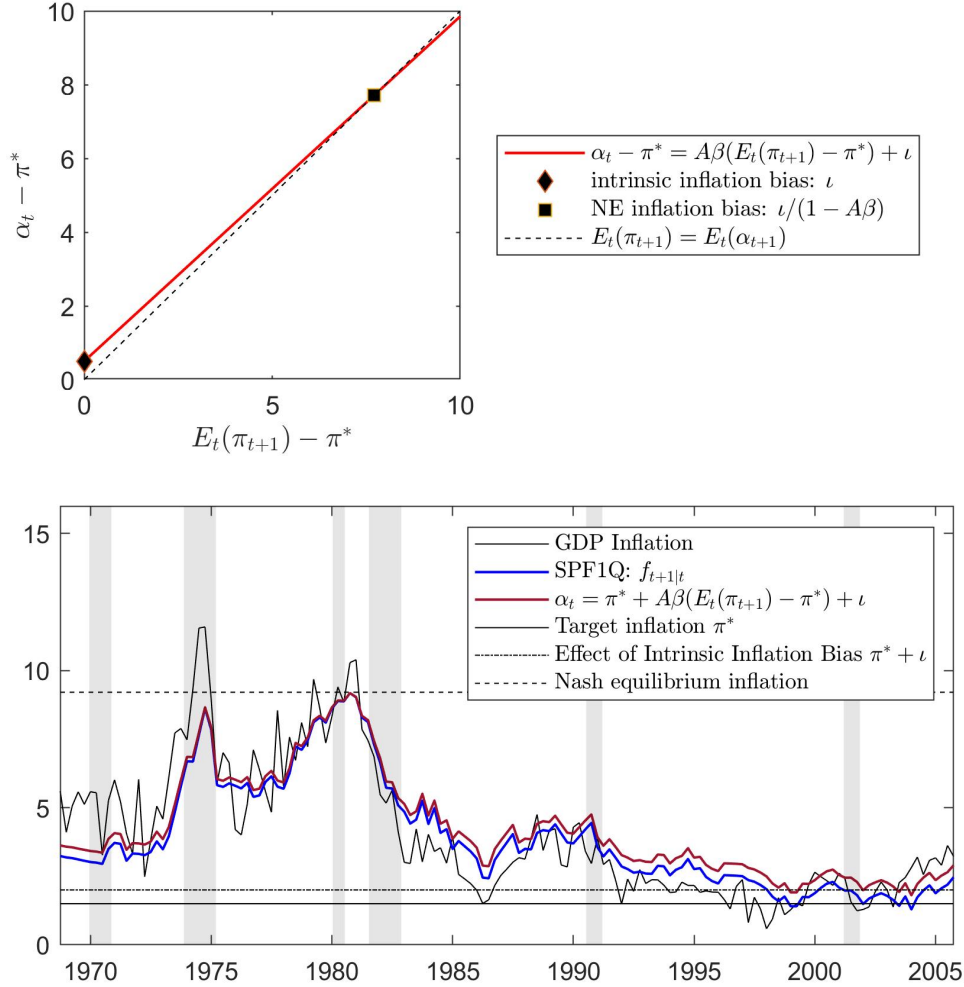
$$303 \quad (9) \quad \alpha^{NE} - \pi^* = \frac{\iota}{1 - A\beta},$$

304 when $\varsigma = 0$: this is greater than the intrinsic bias, particularly when $A\beta$ is
 305 close to 1. To draw the figure, we use two conventional parameter values,
 306 $\pi^* = 1.5\%$ and $\beta = .995$. Since [Blinder \(1997\)](#) sees little intrinsic bias, we set
 307 $\iota = .5\%$. Finally, $A = .94$ leads to a NE bias of 8%.

308 Next, we construct opportunistic intended inflation α using the same pa-
 309 rameters and the SPF one-quarter-ahead forecast as a measure for expected
 310 inflation. The bottom panel of Figure 4 shows how this constructed policy
 311 comoves with expected inflation, along with actual inflation as a reference
 312 point.²⁵ Our analysis of the rise, fall and stabilization of US inflation incorpo-
 313 rates this mechanism.

²⁵The gradual increase in e leading to a gradual increase in α is in line with a discussion
 in [Sargent and Soderstrom \(2000\)](#).

Figure 4: Optimal Response of Opportunistic Policy to Inflation Expectations



In absence of cost push shocks, the intended inflation α_t of the opportunistic policymaker includes a long-run inflation target π^* , an intrinsic inflation bias ι and a response to the private sector's expected inflation $E_t(\pi_{t+1})$. Top panel: The best response function of α_t to the expected inflation $E_t(\pi_{t+1})$. Bottom panel: The time series of α_t constructed using SPF one quarter forecast as a measure for the private sector's expected inflation $E_t(\pi_{t+1})$.

314 4.2.4 Sequential rationality of the committed type

315 The committed policymaker selects and announces a state-contingent plan for
 316 current and future intended inflation $\{a_t\}_{t=0}^{\infty}$ at the beginning of his term and
 317 then subsequently executes it. As the announcement is public information,

318 the committed type has strategic power over the private sector's expectations
319 and thereby over the opportunistic type's intended inflation. In particular,
320 in selecting his state-contingent plan, the committed type takes into account
321 that (i) the private sector's expectation $e(h_t)$ is based on a *consistent* belief
322 system (6), through which the intended inflation strategies of the committed
323 and opportunistic policymaker $a(h_t)$ and $\alpha(h_t)$ determine how e_t responds to
324 the past history h_t ; (ii) the opportunistic intended inflation strategy $\alpha(h_t)$
325 is sequentially rational, satisfying (8), so that it is affected by the expected
326 inflation $e(h_t)$, and in turn by the committed intended inflation strategy $a(h_t)$.

327 The strategy of the committed type is *sequentially rational* if it maximizes
328 his expected present discounted payoff at the beginning of his term,²⁶

$$329 \quad (10) \quad U_0 = \sum_{t=0}^{\infty} (\beta_1(1-q))^t \sum_{h_t} p(h_t) \underline{u}(a_t, e(h_t), \varsigma_t, \tau_t = 1),$$

330 where $\underline{u}(a, e, \varsigma, \tau = 1) \equiv \int u(\pi, x(\pi, e), \varsigma, \tau = 1) g(\pi|a) d\pi$ is the expected mo-
331 mentary objective with x replaced by $x(\pi, e) = (\pi - e - \varsigma) / \kappa$. Note (10) em-
332 ploys the probability of a specific history $h_t = [\varsigma_t, \pi_{t-1}, h_{t-1}]$ where inflation is
333 generated by the committed type, i.e.,²⁷

$$334 \quad (11) \quad p(h_t) = \varphi(\varsigma_t; \varsigma_{t-1}) g(\pi_{t-1} | a(h_{t-1})) p(h_{t-1})$$

335 combining the likelihood of the shock ς , the likelihood of inflation π given the
336 committed type's decision, and the probability of the previous history.

337 4.3 Public Perfect Bayesian Equilibrium

338 We can now define this dynamic game's Public Perfect Bayesian Equilibrium.

²⁶We assume the committed policymaker maximizes payoffs within his own term, so his discounting includes both the time discount factor β_1 and the replacement probability q .

²⁷There is a slight abuse of notation here by using summation Σ over history to capture the joint effects of continuous distribution of π and discrete Markov chain distribution of ς .

DEFINITION 1. A Public Perfect Bayesian Equilibrium is a set of functions $e(h_t)$, $\rho(h_t)$, $\alpha(h_t)$, $a(h_t)$ and $z(h_t)$ such that:

(i) given the policymaker's strategies, $\alpha(h_t)$, $a(h_t)$, and $z(h_t)$, the private sector's belief function $\rho(h_{t+1})$ is updated according to (4); and its expected inflation function $e(h_t)$ satisfies (6);

339 (ii) given the expected inflation function, $e(h_t)$, the strategy for the opportunistic type policymaker, $\alpha(h_t)$ satisfies (8);

(iii) the strategy for the committed type policymaker, $a(h_t)$, maximizes his expected payoff (10); and

(iv) given $\alpha(h_t)$, $a(h_t)$, and $\rho(h_t)$, the private sector's nowcast of inflation conditional on a replacement, $z(h_t)$, satisfies (7).

340 5 Constructing the Equilibrium

341 Construction of the Public Perfect Bayesian equilibrium is usefully viewed as
342 inner and outer loops of a program. The inner loop builds a within-regime equi-
343 librium $\{e(h_t), \rho(h_t), \alpha(h_t), a(h_t)\}$ taking as given beliefs $z(h_t)$ about the con-
344 sequences of a regime change. However, a Public Perfect Bayesian equilibrium
345 requires beliefs $z(h_t)$ consistent with future regime outcomes, so the outer loop
346 adjusts the z to attain a fixed point between $z(h_t)$ and $\{a(h_t), \alpha(h_t), \rho(h_t)\}$.

347 5.1 The principal-agent approach

348 Solving the within-regime equilibrium may appear a formidable task, due to
349 two dynamic game elements. First, the policymaker and the public are con-
350 nected intertemporally: (i) forward-looking expectations make future policies
351 enter the policymaker's current payoffs, thus affecting his current policy choice
352 and (ii) the policymaker's current choices enter the private sector's belief up-
353 dating, thus affecting its future expectations and in turn the policymaker's
354 policy choices. Second, interactions between the two policymaker types arise
355 via private sector expectations: even though one policymaker type is in charge
356 in each period, an optimal choice depends on what the other type would do
357 since private expectations average across both types' policy choices.

358 To tackle these complications, we recast the construction of the within-
359 regime equilibrium as the solution to a principal-agent problem. As principal,

360 the committed policymaker maximizes (10) by choosing state contingent plans
 361 for his current and future actions and those of two agents, the private sector
 362 and the opportunistic policymaker. Two forms of incentive compatibility (IC)
 363 constraints are relevant: (i) private sector consistent beliefs (4) and rational
 364 expectations (6); and (ii) opportunistic type optimal response to expected
 365 inflation (8). We then develop a recursive form for the principal’s problem.

366 Relative to a standard dynamic principal-agent problem, we encounter an
 367 unusual challenge to constructing a recursive optimization problem for the
 368 principal, in that one agent – the private sector – disagrees with the principal
 369 – the committed policymaker – in its belief about the probability of a specific
 370 history. The private sector *thinks* that current inflation could be generated
 371 by the opportunistic policymaker, as reflected by the second line in expected
 372 inflation (6), whereas the committed policymaker *knows* that current inflation
 373 is generated by his policy choices, as reflected in $p(h_t)$ in the intertemporal
 374 objective (10). Such disagreement in probability beliefs between the principal
 375 and the agent creates difficulty in putting a Lagrangian component associated
 376 with the rational expectation constraint (6) in a recursive form, following the
 377 approach laid out by [Marcet and Marimon \(2019\)](#) and others.²⁸

378 We solve this challenge by a “change of measure”. Attaching a multiplier
 379 $\gamma(h_t)$ and the committed type’s probability of history $p(h_t)$ as weights to the
 380 constraint (6), we form the Lagrangian component as:

$$381 \quad (12) \quad \Psi_0 = \sum_{t=0}^{\infty} (\beta_1(1-q))^t \sum_{h_t} p(h_t) \gamma(h_t) [e_t - e(h_t)],$$

382 Then, we rewrite $E[\cdot | h_t, \tau_t = 2]$ in (6) as $\int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [\cdot] \lambda(\pi_t, a_t, \alpha_t) g(\pi_t | a(h_t)) d\pi_t$
 383 where $\lambda(\pi_t, a_t, \alpha_t) \equiv g(\pi_t | \alpha_t) / g(\pi_t | a_t)$ is the likelihood ratio. This allows us
 384 to express (12) recursively, which leads to the following proposition.²⁹

²⁸See also [Kydland and Prescott \(1980\)](#), [Chang \(1998\)](#) and [Phelan and Stacchetti \(2001\)](#).

²⁹Appendix A provides a detailed derivation of the recursive program.

PROPOSITION 1. Given $z(\varsigma, \rho)$, the within-regime equilibrium is the solution to the following recursive optimization problem, subject to the IC constraint $\alpha = Ae + B(\varsigma)$

$$(13) \quad W(\varsigma, \rho, \mu) = \min_{\gamma} \max_{a, \alpha, e} \{ \underline{u}(a, e, \varsigma, \tau = 1) + (\gamma e + \mu \omega) + \beta_1 (1 - q) \int \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) W(\varsigma', \rho', \mu') g(\pi|a) d\pi \},$$

with $\omega \equiv - \left\{ (1 - q) a + qz(\varsigma, \rho) + \frac{1-\rho}{\rho} [(1 - q) \alpha + qz(\varsigma, \rho)] \right\}$

$$(14) \quad \mu' = \frac{\beta}{\beta_1 (1 - q)} \gamma \rho, \text{ with } \mu_0 = 0$$

$$(15) \quad \rho' = \frac{\rho g(\pi|a)}{\rho g(\pi|a) + (1 - \rho) g(\pi|\alpha)}, \text{ with prob } g(\pi|a) \text{ given } \rho_0$$

In this program, some components are familiar from prior research using the recursive approach,³⁰ but some are not due to the unique features of our model. The component $(\gamma e + \mu \omega)$ arises from the forward-looking rational expectations constraint (6). The pseudo state variable μ records past promises made by the committed type in his management of expectations. Next period's pseudo state μ' evolves according (14), keeping track of the shadow price of current promise γ and the effect of a on past expected inflation measured by ρ , as well as adjusting for differing discount factors. A new policymaker is not held accountable for his predecessor's promises, so the initial value of μ is zero.

With two policymaker types and stochastic replacement, the term ω con-

³⁰In fact, if $q = 0$, $\beta_1 = \beta$, and $\rho = 1$ always, this is a textbook NK policy problem in recursive form. For example, in Clarida et al. (1999), the policymaker maximizes $E_0 \sum_{t=0}^{\infty} \beta^t u(\pi_t, x_t)$ subject to $\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \varsigma_t$. To create a dynamic Lagrangian one attaches $E_0 \sum_{t=0}^{\infty} \beta^t \gamma_t [\pi_t - \kappa x_t - \beta E_t \pi_{t+1} - \varsigma_t]$ to the objective. the law of iterated expectation and rearrangement of terms allow this expression to be written as $E_0 \sum_{t=0}^{\infty} \beta^t \{ (\gamma_t - \gamma_{t-1}) \pi_t - \gamma_t \kappa x_t - \gamma_t \varsigma_t \}$ with $\gamma_{-1} = 0$. Defining the pseudo state variable $\mu_t = \gamma_{t-1}$, the recursive optimization along Marcet and Marimon (2019) lines is

$$W(\varsigma_t, \mu_t) = \min_{\gamma_t} \max_{\pi_t, x_t} \{ u(\pi_t, x_t) + \gamma_t (\pi_t - \kappa x_t - \varsigma_t) - \mu_t \pi_t + \beta E_t W(\varsigma_{t+1}, \mu_{t+1}) \}$$

with $\mu_{t+1} = \gamma_t$ and $\mu_0 = 0$. The presence of both min and max stems from the fact that the optimum is a saddlepoint as in the Kuhn-Tucker theorem.

396 tains more than the promised a because private sector’s expected inflation
 397 also depends on the opportunistic policymaker’s intended inflation α and the
 398 inflation z in a new regime. The weights attached to a , α , and z reflect the
 399 exogenous replacement probability q , the endogenous reputation state ρ , and
 400 the divergent probability beliefs about inflation π held by the committed pol-
 401 icymaker and the private sector. This final feature leads to $(1 - \rho)/\rho$ in ω .³¹

402 5.2 The PBE fixed point requirement

403 In a PBE, the nowcast of inflation $z(\varsigma, \rho)$ in a new regime must satisfy

$$404 \quad (16) \quad z^*(\varsigma, \rho) = \rho_0 a^*(\varsigma, \rho_0, 0; z^*(\varsigma, \rho)) + (1 - \rho_0) \alpha^*(\varsigma, \rho_0, 0; z^*(\varsigma, \rho))$$

405 with $a^*(\cdot)$ and $\alpha^*(\cdot)$ obtained from the recursive program (13) given $z^*(\varsigma, \rho)$,
 406 $\rho_0 = l(\rho)$ being partially inherited initial reputation, and $\mu_0 = 0$ as prior
 407 commitments are no longer binding in a new regime.³²

408 5.3 Focusing on policy trade-offs and computation

409 The recursive program in Proposition 1 is valuable, as it sheds light on the
 410 relevant state variables. But it is inefficient for computation because there are
 411 many choice variables. Further, it can be hard to isolate the key trade-offs
 412 facing the policymaker. The following Lemma provides both computational
 413 and conceptual benefits, by developing implications of the forward-looking
 414 rational expectation constraint (6).³³

LEMMA 1. Given (ς, ρ) and that future policymakers follow the equilibrium
 415 strategies: $a^*(\varsigma', \rho', \mu')$, $\alpha^*(\varsigma', \rho', \mu')$ and $z^*(\varsigma', \rho')$, rationally expected in-
 flation $e(\delta, \mu'; \varsigma, \rho)$ is uniquely determined by the contemporaneous policy
 difference $\delta = a - \alpha$, and the future pseudo-state variable μ' .

416 This lemma stems from the fact that the committed policymaker can in-
 417 fluence expected inflation through two channels. Via the *learning channel*, the

³¹Appendix A.9 eliminates the likelihood ratio λ using Bayes’ rule.

³²Schaumburg and Tambalotti (2007) impose such a fixed point requirement in construct-
 ing an equilibrium in which a committed policymaker is randomly replaced.

³³For additional details, see Appendix B.2.

418 committed policymaker affects the future reputation variable ρ' , as a larger pol-
 419 icy difference δ raises the speed of private sector learning about current policy-
 420 maker type. This channel is formalized when we simplify (15) to $\rho' = b(\varepsilon_1, \delta, \rho)$
 421 by replacing $g(\pi|a) = \phi_1(\varepsilon_1)$ and $g(\pi|\alpha) = \phi_2(\pi - a + a - \alpha) = \phi_2(\varepsilon_1 + \delta)$, where
 422 $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are the densities of ε_1 and ε_2 respectively. Via the *expectation*
 423 *anchoring channel*, the committed policymaker adjusts the future pseudo-state
 424 variable, with a higher μ' lowering next-period committed intended inflation
 425 essentially by making it more expensive.

426 Using Lemma 1, we simplify the recursive program (13), moving from
 427 choosing (γ, a, α, e) to merely choosing (δ, μ') . Specifically, we replace e with
 428 $e(\delta, \mu'; \varsigma, \rho)$, α with $Ae + B(\varsigma)$, and a with $\alpha + \delta$ in $\underline{u}(\cdot)$ and $\omega(\cdot)$ of (13) to
 429 obtain $\underline{\underline{u}}(\cdot)$ and $\underline{\underline{\omega}}(\cdot)$ in a simplified program:³⁴

PROPOSITION 2. Given $z^*(\varsigma, \rho)$ and $U^*(\varsigma, \rho, \mu)$, the recursive optimization (13) reduces to

430 (17)
$$W(\varsigma, \rho, \mu) = \max_{\delta, \mu'} \left[\underline{\underline{u}}(\delta, \mu') + \mu \underline{\underline{\omega}}(\delta, \mu') + \beta_1 (1 - q) \Omega(\delta, \mu'; \varsigma, \rho) \right]$$

with $\Omega(\delta, \mu') = \int \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) U^*(\varsigma', b(\varepsilon_1, \delta, \rho), \mu') \phi_1(\varepsilon_1) d\varepsilon_1$.

431 The equilibrium $U^*(\varsigma, \rho, \mu)$ satisfies the following functional fixed point

432 (18)
$$U^*(\varsigma, \rho, \mu) = W(\varsigma, \rho, \mu) - \mu \underline{\underline{\omega}}(\delta^*, \mu'^*)$$

433 where $W(\varsigma, \rho, \mu)$, δ^* and μ'^* are the solution to the simplified recursive pro-
 434 gram (17) conditional on $U^*(\varsigma, \rho, \mu)$.

435 Lemma 1 and Proposition 2 facilitate our computation. With a guessed
 436 function $z(\varsigma, \rho)$ specified in the outer loop, we (i) use $a(\varsigma, \rho, \mu)$, $\alpha(\varsigma, \rho, \mu)$ and
 437 $U(\varsigma, \rho, \eta)$ functions to obtain $e(\delta, \mu'; \varsigma, \rho)$ and $\Omega(\delta, \mu'; \varsigma, \rho)$; (ii) optimize over
 438 (δ, μ') ; (iii) construct new a and α functions from optimal e and δ ; and (iv)
 439 construct new U function. Within the inner loop, we iterate until the policy
 440 functions converge.³⁵ We then calculate a new $z(\varsigma, \rho)$ and repeat the process

³⁴Appendix B provides detailed derivation of this simplified recursive program.

³⁵The problem is not linear quadratic due to Bayesian learning. We therefore use a projection method to obtain a global solution.

441 until the outer loop has reached a fixed point in z .

442 **6 Inflation Regimes and Reputation**

443 We now move to exploring the positive implications of our theory.

444 **6.1 Linking the theory to the data**

445 Quantification requires regime change dates, state variables and parameters.

446 **Regimes:** Choice of possible regime switch dates is subtle. Many mone-
447 tary histories highlight the Fed chair’s identity and nature, as in Friedman
448 and Schwartz’s celebrated Great Contraction chapter. But other histories
449 stress combined efforts of presidential administrations and the central bank
450 (including Meltzer (2014), Levin and Taylor (2013), and Binder and Spin-
451 del (2017)). Our benchmark is to specify a new regime with each chairman:
452 1970Q1 (Burns), 1978Q1 (Miller), 1979Q4 (Volcker’s October 1979 announce-
453 ment of new operating procedures), and 1987Q4 (Greenspan).

454 **Reputation and cost-push shocks:** Proposition 1 highlights three state
455 variables $s_t = [\varsigma_t, \rho_t, \mu_t]$, known to private agents but not to us. We exploit
456 our model’s implication that private agents multi-period inflation forecasts are
457 functions of $E_t(\pi_{t+k}) = f(\varsigma_t, \rho_t, \mu_t, k)$.³⁶ We choose states to exactly match
458 the SPF data $f_{t+k|t}$ at one quarter and three quarter horizons ($k=1,3$)³⁷:

$$459 \quad (19) \quad f_{t+k|t} = E_t(\pi_{t+k}) = f(\varsigma_t, \rho_t, \mu_t, k), \text{ for } k = 1, 3.$$

460 With the extracted state $\hat{\rho}_t$ and the predetermined state $\hat{\mu}_t$, we determine
461 $\hat{\mu}_{t+1}$ using the equilibrium decision rule, $\mu^*(0, \hat{\rho}_t, \hat{\mu}_t)$, continuing recursively
462 to calculate a full history of states.³⁸ At regime switch dates $\hat{\mu}$ is set to zero.

463 Following the term structure intuition discussed earlier, longer-term fore-
464 casts (SPF3Q) depend more on the persistent reputation variable ρ_t , while
465 shorter-term forecasts are more sensitive to transitory price shocks ς_t , as illus-

³⁶Appendix C provides recursive forecasting formulae and state extraction details.

³⁷This allows us to solve for $\hat{\varsigma}_t$ and $\hat{\rho}_t$ given the predetermined pseudo state $\hat{\mu}_t$.

³⁸We use $\mu^*(0, \hat{\rho}_t, \hat{\mu}_t)$ instead of $\mu^*(\hat{\varsigma}_t, \hat{\rho}_t, \hat{\mu}_t)$ so that the extracted $\hat{\varsigma}_t$ will be mean-reverting. Results from using $\mu^*(\hat{\varsigma}_t, \hat{\rho}_t, \hat{\mu}_t)$ are reported in Appendix C.4.

466 trated by the spread between SPF1Q and SPF3Q in Figure 2.³⁹

467 **Parameters:** Table 1 parameters are selected to match some data and to
468 highlight some model mechanisms. The private sector and committed type
469 share a conventional quarterly discount factor based on a 2% annual real rate.
470 The replacement probability of $q = .03$ implies an average regime duration
471 of 8 years. A new policymaker inherits $\rho_0 = .01 + 0.9\rho_{-1}$, implying initial
472 reputation ranging between 1% and 91%.⁴⁰ The 1.5% long-run inflation target
473 lies in the 1 to 2 percent range sometimes cited by central bankers advocating
474 price stability.⁴¹ As in Section 4.2.3, we posit a small intrinsic inflation bias,
475 $\iota = .5\%$ annual rate and set the reduced form parameter $A = .94$ to produce
476 a Nash Equilibrium inflation bias of 8% annual rate at the peak of the Great
477 Inflation. The PC slope κ relates the output gap x to the quarterly inflation
478 π , holding expected inflation fixed, so that $\kappa = .08$ means that an output gap
479 of 3% leads to annualized inflation of -1%, a value compatible with diverse
480 empirical evidence.⁴² Given $A = \vartheta_x / (\vartheta_x + \kappa^2)$, matching $A = .94$ requires
481 $\vartheta_x = 0.1$, within the range used by prominent Fed researchers.⁴³ Finally, all
482 these parameter choices imply that the opportunistic type’s target output gap
483 is $x_2^* = 1.75\%$.⁴⁴

484 Beginning in the 1970s, many studies of inflation use an observable “Food
485 and Energy price shock” (FE shock hereafter).⁴⁵ We initially used this proxy

³⁹We do not use SPF4Q due to missing observations, particularly important in 1975.

⁴⁰This inheritance mechanism would capture private agents rational expectation if a new policymaker’s type is the same as his predecessor with probability .9 and is otherwise a random draw with the chance of a committed type equal to 10%.

⁴¹See Shapiro and Wilson (2019).

⁴²U.S. data from the 1950s and 1960s suggests that a 1% decrease in unemployment led to about 0.54% - 0.65% increase in inflation. An estimate for Okun’s coefficient is about 1.67 using U.S. data prior to 2008, implying a 1% increase in unemployment led to a 1.67% decrease in output. In a structural NKPC, the parameter is also consistent with an adjustment hazard leading to four quarters of stickiness on average and an elasticity of marginal cost with respect to output of unity.

⁴³Brayton et al. (2014) and Orphanides and Williams (2013) after translating time units and using Okun’s law.

⁴⁴Section 4.2.3 links x_2^* to other parameters via $\iota = A(\kappa x_2^* - (1 - \beta)\pi^*)$. With an Okun’s law coefficient of 1.67, $x_2^* = 1.75\%$ is targeting unemployment about 1% below the natural rate. A x_1^* is set slightly below x_2^* for computational reasons.

⁴⁵See R.J. Gordon (2013) and Watson (2014). It is constructed as the difference between the growth rate of the overall personal consumption deflator and its counterpart excluding

Table 1: Parameters

β, β_1	Discount factor (private, committed type)	0.995
q	Replacement probability	0.03
ρ_0	Initial reputation after replacement	$1\% + 0.9\rho_{-1}$
κ	PC output slope	0.08
π^*	Inflation target	1.5%
ϑ_x	Output weight	0.1
x_1^*	Committed type's output target	1.7%
x_2^*	Opportunistic type's output target	1.75%
ν	Persistence of cost-push shock (not δ)	0.7
σ_ξ	Std of cost-push innovation	0.7%
σ_ε	Std of implementation error ε_1 and ε_2	1.2%

One period is a quarter. Inflation target π^* , std of cost-push innovation σ_ξ , and std of implementation error σ_ε are all annualized rates.

486 for ς , but eventually settled on extracting shocks from the SPF because these
 487 real time forecasters appear to better capture various events including the 1974
 488 inflation peak.⁴⁶ The FE shock's serial correlation and its standard deviation
 489 are never the less used to determine ν and σ_ξ . We also combined the FE
 490 shock and the SPF1Q in an initial approximation to the opportunistic intended
 491 inflation α , generalizing the approach behind Figure 4, to obtain the standard
 492 deviation of $(\pi - \alpha)$ that prevailed during 1964Q4-1979Q2 and use it as our
 493 calibrated standard deviation of implementation errors.

494 6.2 Inflation history and model-based inflation policies

495 With an extracted state history, we can construct the *intended inflation policy*
 496 *measures* $\hat{a}_t = a(\hat{s}_t)$ and $\hat{\alpha}_t = \alpha(\hat{s}_t)$. Then, given observed inflation π_t , (2)
 497 implies empirical implementation errors, $\hat{\varepsilon}_{1t} = \pi_t - a(\hat{s}_t)$ and $\hat{\varepsilon}_{2t} = \pi_t - \alpha(\hat{s}_t)$.

498 Figure 5 plots these model-based policies (\hat{a}_t in green, $\hat{\alpha}_t$ in red), and their
 499 associated implementation errors ($\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ with dash-dotted lines in match-
 500 ing color), with regime switch dates marked by solid vertical lines. Within our
 501 model, \hat{a}_t and $\hat{\alpha}_t$ are private agent beliefs about the intended inflation of each

food and energy. We display its time series in Appendix C.5.

⁴⁶For additional discussion, see Appendix C.5

502 policymaker type and are constructed using states extracted from the SPF
503 forecasts conditional on the set of regime switch dates. Actual inflation π is
504 the black dashed line, but recall that it doesn't enter construction of \hat{a}_t or $\hat{\alpha}_t$.

505 Seeing the entire history of these series, we have an advantage relative to
506 private agents: they only know events through date t . Using this advantage, we
507 identify the Burns-Miller interval 1971Q1-1979Q2 as an opportunistic regime
508 because (i) $\hat{\varepsilon}_2 = \pi - \hat{\alpha}$ fluctuates around 0, suggesting that actual inflation
509 consistent with opportunistic policy, and (ii) $\hat{\varepsilon}_1 = \pi - \hat{a}$ is generally positive,
510 suggesting actual inflation inconsistent with committed policy.

511 By the 1982Q4 recession, the situation is clearly reversed. Actual inflation
512 more closely resembles committed policy, with $\hat{\varepsilon}_1 = \pi - \hat{a}$ fluctuating around
513 zero, whereas actual inflation lies below opportunistic policy, with $\hat{\varepsilon}_2 = \pi - \hat{\alpha}$
514 being generally negative. We thus identify the bulk of the Volcker regime
515 and the full Greenspan regime as involving commitment policy. Perhaps more
516 controversially,⁴⁷ Figure 5 breaks the history at the start of the Reagan admin-
517 istration in 1981Q1 (marked by a dashed vertical line): the average mean of
518 $\hat{\varepsilon}_2 = \pi - \hat{\alpha}$ is 0.13% in the earlier interval, and the average mean of $\hat{\varepsilon}_1 = \pi - \hat{a}$
519 is only 0.019% in the later interval. By contrast, the average means of $\hat{\varepsilon}_1$ in
520 the earlier interval and $\hat{\varepsilon}_2$ in the later interval are 1.5% and -1%, respectively.

521

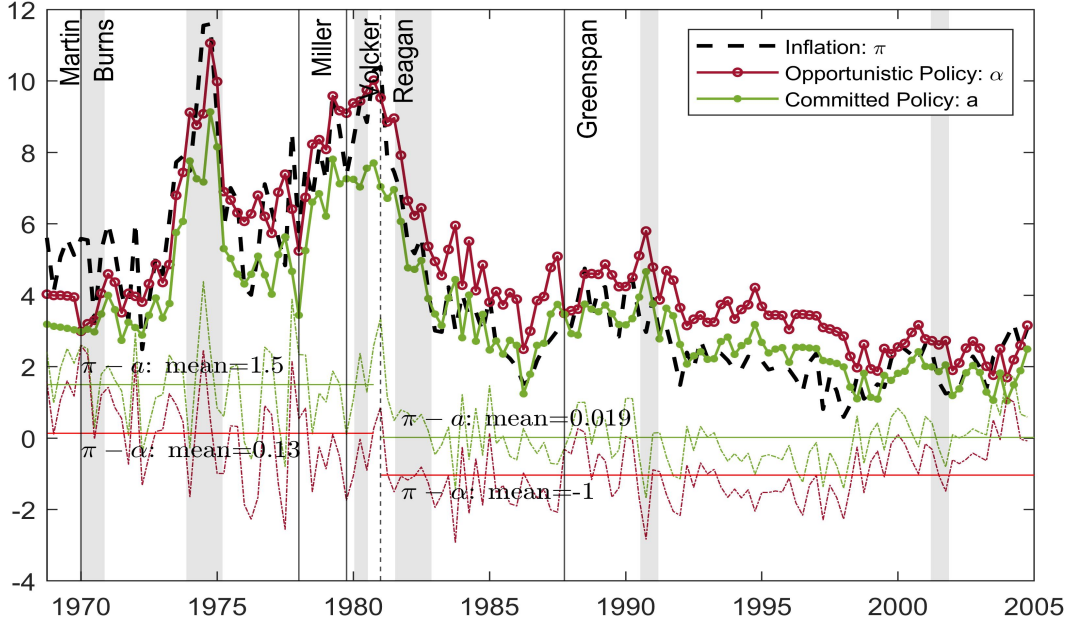
522 6.3 Interpreting US inflation history 1968-2005

523 We now examine US inflation history assuming there is an opportunistic pol-
524 icymaker early on and a committed policymaker later. At any point in time,
525 one policy is the current policymaker's intended inflation and the other policy
526 is private agents' rational belief about an alternative policymaker's behavior
527 if confronted with the same observable history.

528 Figure 6 shows our model-based interpretation of US inflation history 1968-
529 2005. Three time series – inflation π (black), model-implied committed policy
530 \hat{a} (green), and model-implied opportunistic policy $\hat{\alpha}$ (red) are repeated from
531 Figure 5. But before 1981Q1, the red line is solid and the green line is dotted
532 because an opportunistic policymaker is taken to be generating the observed
533 inflation. After 1981Q1, the red line is dotted and the green line is solid as a

⁴⁷See Goodfriend and R.G. King (2005) and Orphanides (2005)

Figure 5: Inflation history and model-implied policies



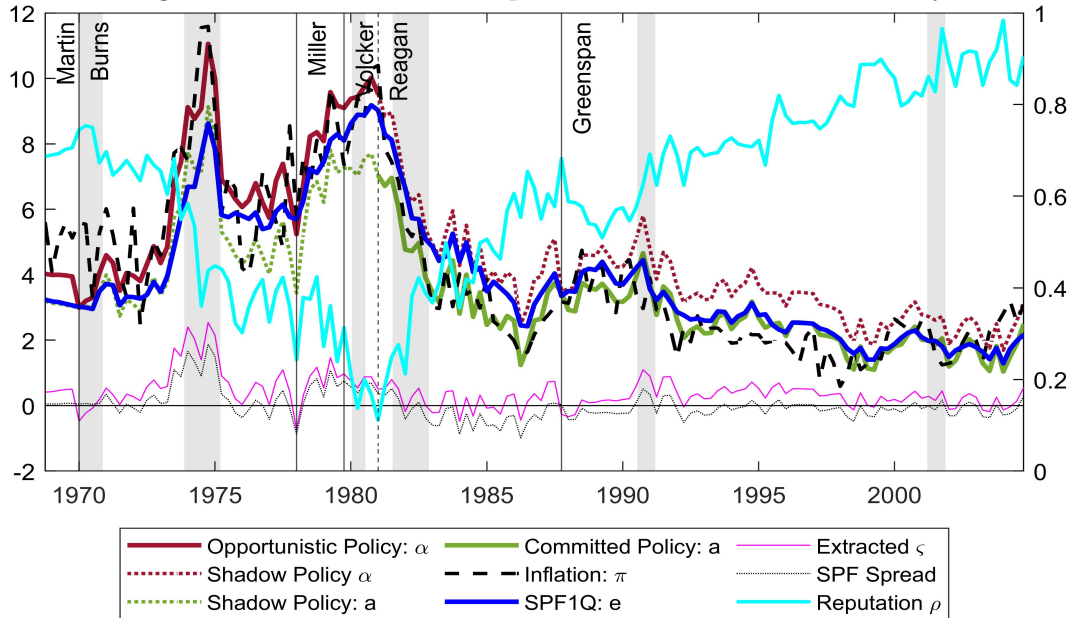
534 committed policymaker is generating the observed inflation. Figure 6 also plots
 535 the SPF1Q forecast (blue), exactly matched by our model expected inflation.

536

537 Our framework thus sheds light on why inflation forecast errors turned from
 538 persistently positive to persistently negative around 1980, as highlighted by
 539 Figure 1. Opportunistic intended inflation α is always higher than committed
 540 intended inflation a in our model and expected inflation e is roughly a weighted
 541 average of the two. Observed inflation before 1981Q1 is tracked by our oppor-
 542 tunistic policy measure $\hat{\alpha}$ so it exceeds expected inflation – the SPF1Q, hence
 543 persistently positive inflation forecast errors arise. After 1981Q1, observed in-
 544 flation is instead tracked by our committed policy measure \hat{a} , lying below the
 545 SPF1Q, yielding persistently negative inflation forecast errors.

546 Figure 6 also plots the extracted cost-push shock $\hat{\zeta}$ (magenta) and the
 547 extracted reputation state $\hat{\rho}$ (cyan and measured on the right hand axis).
 548 Note first that the extracted cost-push shock $\hat{\zeta}$ covaries strongly with the SPF
 549 spread (SPF1Q-SPF3Q plotted in black dotted line), consistent with state

Figure 6: Model-based interpretation of US inflation history



550 extraction exploiting greater sensitivity of near-term forecasts to transitory
 551 shocks. Note next the extracted reputation's big swing: $\hat{\rho}$ starts from .7 in
 552 1968, decreases through the 1970s to a 1981Q1 trough at .1, and increases
 553 afterwards to above .9 in 2005. These reputation dynamics are quantitatively
 554 important for our model-implied policy measures, as we will show next.

555 7 Reputation and Policy

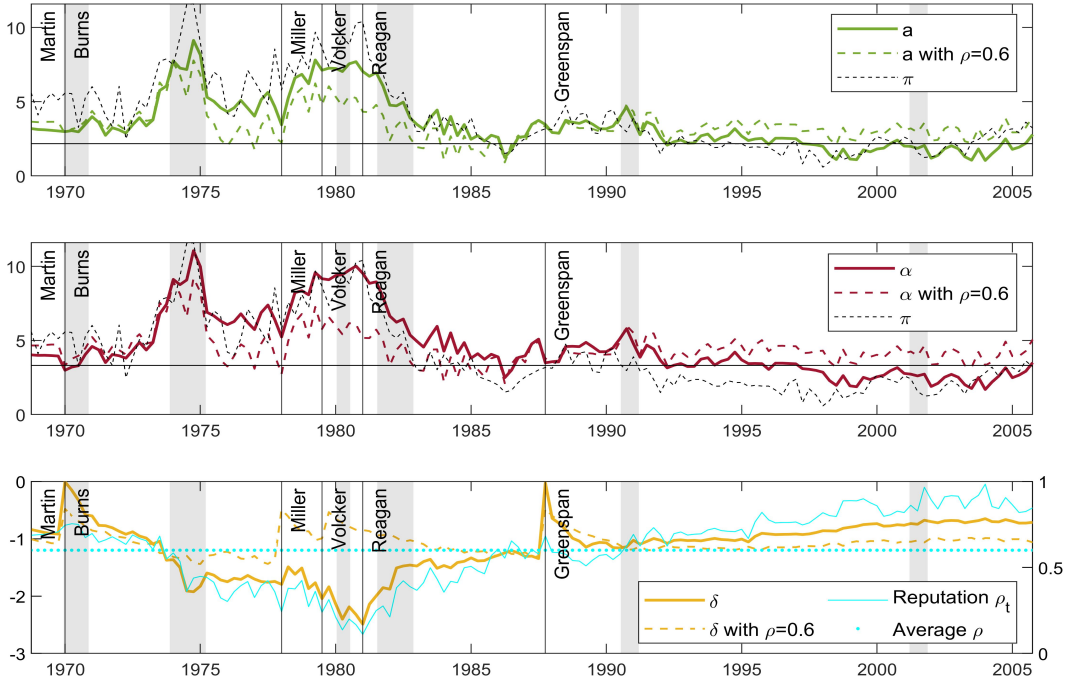
556 We start with a historical decomposition illustrating the quantitative impor-
 557 tance of time-varying reputation in aligning our model-implied policies with
 558 historical inflation. We then use equilibrium decision rules to explain how rep-
 559 utation affects optimal policies, and in the process, highlight the crucial role of
 560 having a purposeful non-committed policymaker to time-varying reputation.

561 7.1 Historical Decomposition

562 Our framework permits a *historical decomposition* of \hat{a} and $\hat{\alpha}$ into parts at-
 563 tributable to each state (ζ, ρ, μ) . To focus on the importance of the reputation

564 state ρ for optimal policies, we construct new policy measures with reputation
 565 at a reference value $\underline{\rho}$, but maintaining the cost-push shock $\widehat{\zeta}_t$ and the pseudo-
 566 state variable $\widehat{\mu}_t$ at extracted levels. The gap between constant- ρ and original
 567 policies measures the effect of time-varying reputation.⁴⁸ Figure 7 contrasts
 568 these constant- ρ policies (dash-dotted line) constructed using $(\widehat{\zeta}_t, \underline{\rho}, \widehat{\mu}_t)$, with
 569 original policies (solid line) constructed using $(\widehat{\zeta}_t, \widehat{\rho}_t, \widehat{\mu}_t)$. Observed inflation
 570 (black line) facilitates assessments on how much time variation in ρ helps our
 571 model match the US inflation experience.

Figure 7: Historical decomposition: effect of ρ constant at historical average.



572 The top two panels in Figure 7 reveal just how important time-varying ρ is
 573 for the intended inflation measures \widehat{a} and $\widehat{\alpha}$. Between 1974Q1 and 1985Q4, the
 574 constant- ρ policies lie below the original policies, with particularly large gaps
 575 during the Great Inflation and the Volcker Disinflation. Our model would

⁴⁸We hold the reputation state constant at $\underline{\rho} = 0.6$ – the sample average of extract reputation state $\widehat{\rho}_t$ during 1968Q4-2005Q4.

576 badly miss these two important episodes of inflation history without time-
577 varying reputation, even if equipped with the same price shocks and regime
578 changes. When inflation is relatively stable after 1990, our model-based poli-
579 cies are quite close to observed inflation, but the constant- ρ policies are uni-
580 formly greater. That is, our model would miss the Great Moderation too if it
581 omitted time-varying reputation.

582 Section 5.3 showed how to formulate the committed type’s choice problem
583 in terms of the policy difference $\delta = a - \alpha$, which is key to Bayesian learning
584 and expected inflation. The model-implied policy difference $\widehat{\delta} = \widehat{a} - \widehat{\alpha}$ (solid
585 line) and its constant- ρ counterpart (dash-dotted line),⁴⁹ are shown in the
586 bottom panel of Figure 7 along with the reputation state $\widehat{\rho}$ (cyan solid line)
587 and its historical average (cyan dotted line) measured on the right hand axis.
588 Three notable features shed light on the large gaps between constant- ρ policies
589 and original policies (a, α) in the top two panels. First, the policy difference $\widehat{\delta}$
590 moves closely with the extracted reputation state $\widehat{\rho}$. Second, when $\widehat{\rho}$ is lower
591 than its historical average, e.g., between 1974Q1 and 1985Q4, both \widehat{a} and $\widehat{\alpha}$ rise
592 above their constant- ρ counterparts, with the policy difference $\widehat{\delta}$ larger than
593 its constant- ρ counterpart. Third, when $\widehat{\rho}$ is higher than its historical average,
594 e.g., after 1990, both \widehat{a} and $\widehat{\alpha}$ fall relative to their constant- ρ counterparts,
595 with the policy difference $\widehat{\delta}$ smaller than its constant- ρ counterpart.

596 7.2 Effects of reputation on equilibrium decision rules

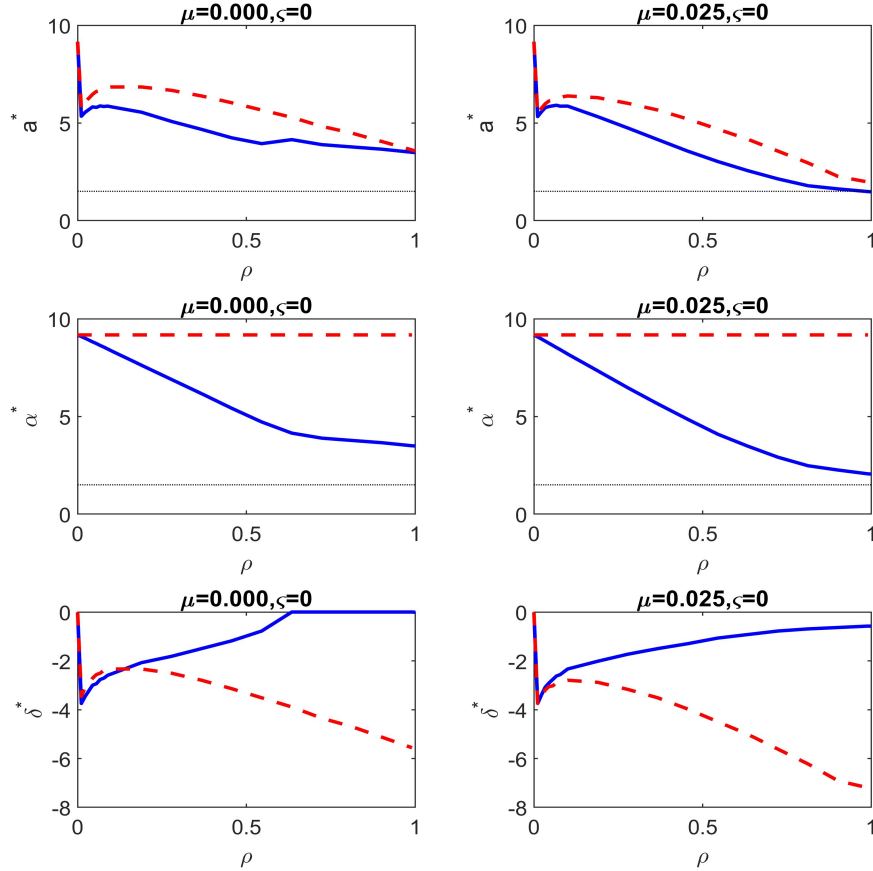
597 Equilibrium decisions $\{a^*, \alpha^*\}$ and their difference $\delta^* = a^* - \alpha^*$ depend on
598 reputation ρ , along with the other two more standard states. The three panels
599 of Figure 8 display how each choice (blue line) depends on ρ , conditional on
600 the cost-push shock ς being zero and two levels of the pseudo state μ .⁵⁰ In
601 the top and middle panels, we also plot the inflation target $\pi^* = 1.5\%$ (black
602 dotted line): note that a^* in the upper right panel is at this level when $\rho = 1$.

603 The middle panels highlight that the equilibrium opportunistic policy α^*
604 decreases with reputation ρ , which is intuitive given that higher reputation

⁴⁹ $\delta(\widehat{\varsigma}_t, \rho, \widehat{\mu}_t) = a(\widehat{\varsigma}_t, \rho, \widehat{\mu}_t) - \alpha(\widehat{\varsigma}_t, \rho, \widehat{\mu}_t)$ is the constant- ρ policy difference.

⁵⁰The two values are zero (the initial value at the regime switch date) and the certainty steady state value when $\rho = 1$. We choose these levels because most equilibrium values of μ lie between them in absence of the cost-push shock.

Figure 8: Effects of ρ on equilibrium policies



Equilibrium decision rules: top panels: intended inflation of committed policymaker a^* ; middle panels: intended inflation of non-committed policymaker α^* ; bottom panels: policy difference $\delta^* = a^* - \alpha^*$. Blue solid lines are decision rules in our model where the non-committed policymaker optimally responds to the expected inflation. Red dashed lines are decision rules in a model where the non-committed policymaker mechanically adopts a policy rule that would be optimal if $\rho = 0$.

605 yields lower expected inflation and the Section 4.2.3 link between α^* and e .

606 The consequences of reputation for the policy difference, δ^* , are shown in
 607 the bottom panels. Notice first that $\delta^* \leq 0$: equilibrium committed policy is
 608 always lower than equilibrium opportunistic policy. Intuitively, the committed
 609 policymaker invests in reputation when $\rho < 1$. Notice next that δ^* increases

610 with ρ for $\rho > 0$, indicating diminishing returns to investing in reputation.⁵¹
 611 Third, at high reputation, δ^* is either zero or close to zero.⁵² Consequently,
 612 the learning speed of private agents is zero or very slow when the policymaker
 613 is likely to be the committed type, because the observed inflation is only
 614 informative about type when the two types behave differently.

615 The equilibrium committed policy a^* , shown in the top panels, can be
 616 understood as the sum of α^* and δ^* . Therefore, the effect of reputation ρ
 617 on a^* depends on the relative strength of effects on α^* versus on δ^* . In our
 618 calibration, the Nash Equilibrium inflation bias (α^* at $\rho = 0$) is much higher
 619 than the intrinsic inflation bias (α^* at $\rho = 1$), resulting in a dominant effect of
 620 ρ on α^* . In turn, a^* is generally decreasing in ρ , with a flatter slope than α^* .

621 These decision rules help us understand the historical decomposition in
 622 Figure 7: since a^* and α^* are both decreasing in ρ , our model-implied \hat{a} and
 623 $\hat{\alpha}$ are higher than their constant- ρ counterparts when extracted $\hat{\rho}$ is below its
 624 historical average, and vice versa.

625 An important new element, relative to our prior work (Lu et al. (2016)),
 626 is a purposeful, if myopic, policymaker rather than a mechanical alternative
 627 type. If we instead assume that the non-committed policymaker mechanically
 628 adopts a policy rule that would be optimal if $\rho = 0$ – incorporating the Nash
 629 Equilibrium inflation bias – then matters are very different: the results are
 630 the red dashed lines in Figure 8. The most salient implication is for the
 631 policy difference δ^* . Comparing the red dashed lines with the blue lines in the
 632 bottom panels, we find that at majority values of ρ , the policy difference is
 633 much larger than when the non-committed policymaker is purposeful. With
 634 such a mechanical alternative policymaker, the large δ^* means that private
 635 agents learn about policymaker type so fast that we lose the time-varying
 636 reputation shown above to crucial for capturing many elements of the US
 637 inflation experience.⁵³

⁵¹Moreover, it becomes harder to distinguish between the two policy regimes when the private sector attaches a higher likelihood that it is the committed policymaker in place.

⁵²That is, small relative to 1.2% standard deviation of ε_1 and ε_2 .

⁵³Recall the standard deviation of implementation error in our calibration is 1.2%. When the equilibrium policy difference δ^* is as large as three or four times 1.2%, as the red line indicates at majority values of ρ , the policymaker’s type will be revealed immediately.

8 Exploring Credibility and Counterfactuals

Our framework sheds light on the much-discussed idea of credibility and permits us to undertake an important counterfactual.

8.1 Credibility and Reputation

Macroeconomists frequently discuss policymaker reputation, as we have above, and the credibility of a specific announcement or program,⁵⁴ as we have not. As a prelude to a survey of macroeconomists and central bankers about credibility, [Blinder \(2000\)](#) remarks that his “own favorite definition involves matching deeds to words: a central bank is credible if people believe it will do what it says.” While we also like this definition,⁵⁵ it is incomplete because it does not allow for partial, but not perfect, credibility.⁵⁶ Practical macroeconomists and central bankers regularly discuss ideas such as “greater credibility improves the short-run inflation-unemployment trade-off,” “greater credibility brings down the cost of reducing inflation” and “once low inflation has been achieved, a more credible central bank is better able to maintain low inflation.”⁵⁷ We now describe two measures of partial credibility of a committed policymaker’s policy plan $a(s)$ and display these along with reputation in [Figure 9](#).

Credibility gap in inflation units One intuitive measure is the distance between the $a(s)$ and the private sector’s nowcast of inflation $E(\pi|s)$,⁵⁸ i.e.,

$$(20) \quad a(s) - E(\pi|s) = (1 - \rho)[a(s) - \alpha(s)] = (1 - \rho)\delta.$$

so that it depends only on reputation and the policy difference δ .

⁵⁴For example, some point to a country’s long-term interest rate, presuming it dominated by inflation expectations, as a measure of credibility for low inflation. [M. King \(2005\)](#) interprets international cross-section of nominal rates in this way. He highlights shifts in nominal and real yields during notable U.K. events, while [Goodfriend \(1993\)](#) links U.S. long-term nominal interest rate to inflation scares, and evolving credibility.

⁵⁵See the opening discussion of “Managing Expectations” ([R.G. King et al. \(2005\)](#))

⁵⁶As [M. King \(2005\)](#) puts it “credibility is not an all-or-nothing matter. Policy is neither credible nor incredible. It is, as we say in economics, a continuous variable.”

⁵⁷These quotes are from [Blinder \(2000\)](#), p. 145.

⁵⁸[Cukierman and Meltzer \(1986\)](#) define credibility as: “the absolute distance between the policymaker’s plans and agents beliefs about those plans.” If $a=0$ and $\alpha > 0$ is constant, $a(s) - E(\pi|s)$ depends only on ρ and varies inversely with it.

659 **Degree of credibility** In an inflation targeting context, credibility is some-
660 times related to the private sector’s probability that inflation will fall in a
661 band around the target, e.g., $a - \theta \leq \pi \leq a + \theta$. In our setup, this probabil-
662 ity reflects implementation errors and the private sector’s lack of knowledge
663 about policymaker type.⁵⁹ We now assume normal implementation errors and
664 let $N(\cdot, \bar{\pi}, \sigma)$ be the normal cdf with mean $\bar{\pi}$ and standard deviation σ . Our
665 second credibility measure is

$$666 \quad (21) \quad \psi(a, \alpha, \theta, \rho, \sigma) = \rho + (1 - \rho) \frac{[N(a + \theta, \alpha, \sigma) - N(a - \theta, \alpha, \sigma)]}{[N(a + \theta, a, \sigma) - N(a - \theta, a, \sigma)]}$$

667 which is the ratio of the private sector’s probability that inflation falls within
668 the band relative to the committed policymaker’s probability. Note that the
669 denominator expression is constant across $a(s)$, while the numerator may be
670 written to stress the policy difference, $N(\delta + \theta, 0, \sigma) - N(\delta - \theta, 0, \sigma)$. That
671 is, our second credibility measure also depends on reputation ρ and the policy
672 difference δ . Figure 9 displays this credibility measure for $\theta = \sigma$, along with the
673 credibility gap and reputation. There is a strikingly high correlation between
674 the credibility gap in inflation units and the degree of credibility.

675 **Credibility and reputation** Both of these evolving, partial credibility mea-
676 sures depend on near-term inflation. Under commitment, though, a long-
677 lasting regime will attain $\rho = 1$ and intended inflation will have a stationary
678 distribution with $E(a) = \pi^*$. Hence, ρ_t is a measure of longer-term credi-
679 bility and, in particular, of the date t likelihood that the current regime will
680 achieve “price stability.” In this sense, our model captures the views of some
681 academicians in the [Blinder \(2000\)](#) survey: “a central bank can raise the pub-
682 lic’s subjective probability that it is ‘tough’ by keeping inflation low. This
683 probability is, in turn, taken as a measure of the bank’s credibility.” It is also

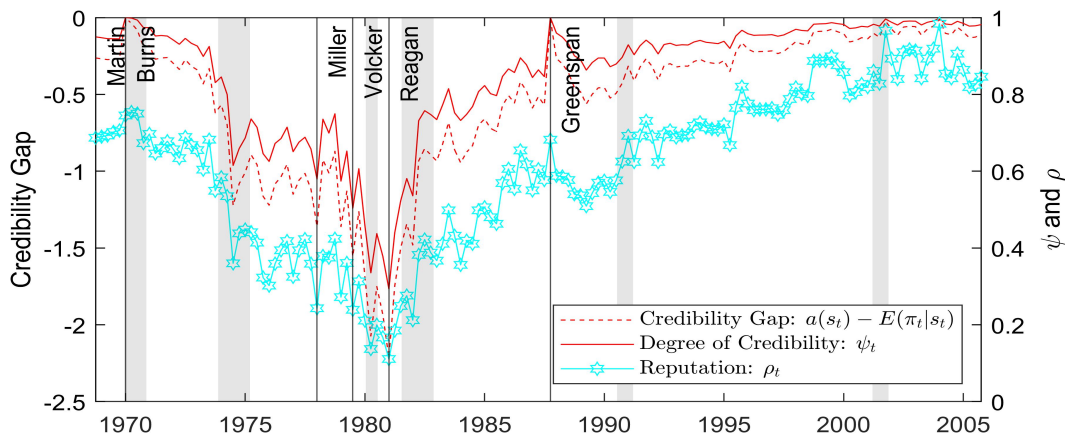
⁵⁹Normal errors imply the private sector’s probability of $a - \theta \leq \pi \leq a + \theta$ is

$$\int_{a-\theta}^{a+\theta} [\rho n(\pi, a, \sigma) + (1 - \rho)n(\pi, \alpha, \sigma)] d\pi$$

$$= \rho[N(a + \theta, a, \sigma) - N(a - \theta, a, \sigma)] + (1 - \rho)[N(a + \theta, \alpha, \sigma) - N(a - \theta, \alpha, \sigma)].$$

Expressing this as a ratio to $[N(a + \theta, a, \sigma) - N(a - \theta, a, \sigma)]$ leads to (21). This measure is readily generalized to an asymmetric band and type-specific implementation error volatility.

Figure 9: Credibility and Reputation



Two measures of the short-term policy credibility are closely associated: the inflation credibility gap defined as $a(s) - E(\pi|s) = (1 - \rho)[a(s) - \alpha(s)] = (1 - \rho)\delta$ and the degree of credibility defined as the ratio of the private sector’s probability that $a - \theta \leq \pi \leq a + \theta$ relative to the committed type’s probability. Further, these two measures also rise and fall over time with reputation, ρ , which can be viewed as the likelihood that inflation will be at π^* if the current regime continuous for a long time.

684 consistent with his summary “that many central bankers take the degree of
685 dedication to price stability as synonymous with credibility.”

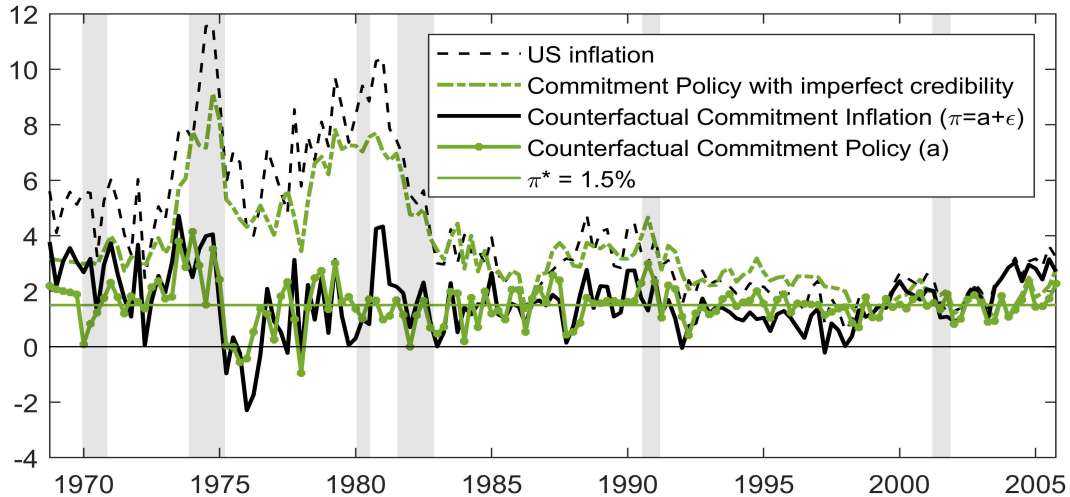
686 8.2 US inflation under a commitment regime

687 We have seen that our theory provides a potential explanation of the behavior
688 of US inflation over 1968 to 2005, with key ingredients being regime shifts,
689 inability of some policymakers to commit, and private sector learning. We now
690 consider how inflation would have evolved if there had been a single committed
691 policymaker in place for the entire period, faced with the same series of ζ and
692 ε shocks, and with his type known by private agents.

693 Figure 10 displays the answer which is striking. The green line labelled
694 “commitment policy” is $\hat{a}^c = a^*(\hat{\zeta}, \rho = 1, \mu)$ so that it evolves with extracted
695 price shocks, while the black solid line marked “commitment inflation” is $\hat{\pi}^c =$
696 $\hat{a}^c + \hat{\varepsilon}$ so that it contains the same extracted implementation errors.⁶⁰ These

⁶⁰ $\hat{\varepsilon}_t = \hat{\varepsilon}_{2,t}$ before 1981Q1 and $\hat{\varepsilon}_t = \hat{\varepsilon}_{1,t}$ afterwards.

Figure 10: Counterfactual Inflation Under Commitment



Counterfactual rates are computed assuming a single committed policymaker in place for the entire period, faced with the same shocks and with his type known by private agents. The counterfactual US inflation is dramatically different from observed inflation, even with large temporary “price shocks” in the 1970s. While the policymaker permits inflation to rise temporarily in response to such shocks, there is a subsequent reversal reflecting the desirability of price level targeting in this New Keynesian model.

697 two series are governed by general NK policy principles. First, our model’s
 698 timing assumptions imply that policy under commitment will not respond to
 699 ε , because these one-time disturbances do not affect intertemporal trade-offs.
 700 Second, in line with the “flexible inflation targeting” analytics of Clarida et
 701 al. (1999), the dramatic price shocks of the 1970s lead to an increase in actual
 702 and intended inflation, rising to about 4% relative to a long-run target π^*
 703 of 1.5%. But this above-average inflation is soon followed by an interval of
 704 inflation below the long-run target and the large price shocks lead to deflation
 705 in 1975-1976.

706 For most of 1968-2005, intended inflation is very different in this full com-
 707 mitment counterfactual than it would have been with a series of committed
 708 policymakers facing evolving reputation (the dashed green line) as in our earlier
 709 historical decomposition. However, after the mid 1990s, there is a relatively
 710 small difference as reputation is at a high level.

9 Conclusions and Final Remarks

We show that a monetary regime shift model can capture the main features of U.S. inflation between the late 1960s and the mid 2000s. Our setup features a standard forward-looking New Keynesian Phillips curve, policymakers of differing commitment capacity, Bayesian learning by the private sector policymaker type, and occasional observable changes in regime.

Both types of policymakers, committed and opportunistic, behave purposefully in line with the earlier 1980s literature on monetary policy and inflation bias. To this end, we construct a Bayesian perfect equilibrium, in which policymakers and private agents rationally anticipate future regime change. Within a regime, a committed policymaker solves a recursive optimization problem that generalizes a now-standard approach in two necessary and important ways. First, the committed policymaker takes into account the effect of policy actions – intended inflation – on reputation, defined as the private sector’s rational belief that a committed type is in place. Second, the committed policymaker understands that (i) private sector inflation expectations include future behavior of an opportunistic type; and (ii) an opportunistic type’s intended inflation depends on private sector inflation expectations. A compact representation of this optimization problem permits calculation of decision rules and construction of time series within the Bayesian perfect equilibrium.

Our framework has state variables, observed by the policymaker and private agents, but not by us. We use the inflation forecasts from the Survey of Professional Forecasters to extract these state variables (reputation and a cost-push shock). These state variables allow us to construct time series of intended inflation for committed and opportunistic policymakers from the SPF data without using actual inflation. Yet, when we assume regimes with opportunistic policymakers before 1981 and regimes with committed policymakers afterward, the corresponding intended inflation tracks US inflation’s rise, fall, and stabilization between 1970 and 2005.

Our model is deliberately stark. But it yields results that have surprised us and others. We believe its success in matching the U.S. time series implies great promise to further research on models that feature agents learning about the commitment capacity of purposeful policymakers within various regimes.

References

- 745 **Atkeson, Andrew and Patrick J. Kehoe**, “The advantage of transparency
746 in monetary policy instruments,” *Federal Reserve Bank of Minneapolis Staff*
747 *Report 297*, 2006.
- 748 **Backus, David A. and John Driffill**, “Inflation and Reputation.,” *Ameri-*
749 *can Economic Review*, 1985, *75(3)*, 530–538.
- 750 — and —, “Rational Expectations and Policy Credibility Following a Change
751 in Regime,” *The Review of Economic Studies*, April 1985, *52 (2)*, 211–221.
- 752 **Barro, Robert J.**, “Reputation in a model of monetary policy with incom-
753 plete information,” *Journal of Monetary Economics*, January 1986, *17 (1)*,
754 3–20.
- 755 — and **David B. Gordon**, “A Positive Theory of Monetary Policy in a
756 Natural Rate Model,” *Journal of Political Economy*, 1983, *91 (4)*, 589–610.
- 757 — and —, “Rules, discretion and reputation in a model of monetary policy,”
758 *Journal of Monetary Economics*, 1983, *12 (1)*, 101–121.
- 759 **Bianchi, Francesco**, “Regime Switches, Agents' Beliefs, and Post-World War
760 II U.S. Macroeconomic Dynamics,” *The Review of Economic Studies*, oct
761 2012, *80 (2)*, 463–490.
- 762 **Binder, Sarah A. and Mark Spindel**, *The myth of independence: how*
763 *Congress governs the Federal Reserve*, Princeton ; Oxford: Princeton Uni-
764 versity Press, 2017.
- 765 **Blinder, Alan S.**, “Distinguished Lecture on Economics in Government:
766 What Central Bankers Could Learn from Academics—and Vice Versa,”
767 *Journal of Economic Perspectives*, May 1997, *11 (2)*, 3–19.
- 768 **Blinder, Alan S.**, “Central-Bank Credibility: Why Do We Care? How Do
769 We Build It?,” *The American Economic Review*, 2000, *90 (5)*, 1421–1431.
- 770 **Brayton, Flint, Thomas Laubach, and David Reifschneider**, “Optimal-
771 Control Monetary Policy in the FRB/US Model,” *FEDS Notes*, November
772 2014, *2014 (0035)*.
- 773 **Chang, Roberto**, “Credible Monetary Policy in an Infinite Horizon Model:
774 Recursive Approaches,” *Journal of Economic Theory*, 1998, *81*, 431 – 461.
- 775 **Chari, V. V. and Patrick J. Kehoe**, “Sustainable Plans,” *Journal of Po-*
776 *litical Economy*, 1990, *98 (4)*, 783–802.

- 777 **Clarida, Richard, Jordi Gali, and Mark Gertler**, “The Science of Mon-
778 etary Policy: A New Keynesian Perspective,” *Journal of Economic Litera-*
779 *ture*, 1999, *37* (4), 1661–1707.
- 780 **Cogley, Timothy, Christian Matthes, and Argia M. Sbordone**, “Op-
781 timized Taylor rules for disinflation when agents are learning,” *Journal of*
782 *Monetary Economics*, may 2015, *72*, 131–147.
- 783 —, **Giorgio E. Primiceri, and Thomas J. Sargent**, “Inflation-Gap Per-
784 sistence in the US,” *American Economic Journal: Macroeconomics*, 2010, *2*
785 (1), 43–69.
- 786 **Coibion, Olivier, Yuriy Gorodnichenko, and Rupal Kamdar**, “The
787 Formation of Expectations, Inflation, and the Phillips Curve,” *Journal of*
788 *Economic Literature*, December 2018, *56* (4), 1447–1491.
- 789 **Cukierman, Alex and Allan H. Meltzer**, “A Theory of Ambiguity, Credi-
790 bility, and Inflation under Discretion and Asymmetric Information,” *Econo-*
791 *metrica*, September 1986, *54* (5), 1099.
- 792 — **and Nissan Liviatan**, “Optimal accommodation by strong policymakers
793 under incomplete information,” *Journal of Monetary Economics*, February
794 1991, *27* (1), 99–127.
- 795 **Debortoli, Davide and Aeimit Lakdawala**, “How Credible Is the Federal
796 Reserve? A Structural Estimation of Policy Re-Optimizations,” *American*
797 *Economic Journal: Macroeconomics*, jul 2016, *8* (3), 42–76.
- 798 — **and Rivcardo Nunes**, “Monetary Regime Switches and Central Bank
799 Preferences,” *Journal of Money, Credit and Banking*, 2014, *46* (8), 1591–
800 1625.
- 801 **DeLong, J. Bradford**, “America’s Only Peacetime Inflation: The 1970s,”
802 Technical Report h0084, National Bureau of Economic Research, Cam-
803 bridge, MA May 1996.
- 804 **Dovis, Alessandro and Rishabh Kirpalani**, “Rules without Commitment:
805 Reputation and Incentives,” *The Review of Economic Studies*, feb 2021.
- 806 **Erceg, Christopher J. and Andrew T. Levin**, “Imperfect credibility and
807 inflation persistence,” *Journal of Monetary Economics*, May 2003, *50* (4),
808 915–944.
- 809 **Eusepi, Stefano and Bruce Preston**, “The Science of Monetary Policy: An
810 Imperfect Knowledge Perspective,” *Journal of Economic Literature*, March
811 2018, *56* (1), 3–59.

- 812 **Evans, George W. and Seppo Honkapohja**, “Expectations, Learning,
813 And Monetary Policy: An Overview Of Recent Research,” Working Papers
814 Central Bank of Chile 501, Central Bank of Chile October 2008.
- 815 **Evans, Martin and Paul Wachtel**, “Inflation Regimes and the Sources of
816 Inflation Uncertainty,” *Journal of Money, Credit and Banking*, 1993, 25 (3),
817 475–511.
- 818 **Faust, Jon and Lars E. O. Svensson**, “Transparency and Credibility:
819 Monetary Policy with Unobservable Goals,” *International Economic Review*,
820 2001, 42 (2), 369–397.
- 821 **Golosov, M., A. Tsyvinski, and N. Werquin**, “Recursive Contracts and
822 Endogenously Incomplete Markets,” in “Handbook of Macroeconomics,” El-
823 sevier, 2016, pp. 725–841.
- 824 **Goodfriend, Marvin**, “Interest Rate Policy and the Inflation Scare Prob-
825 lem: 1979-1992,” *Federal Reserve Bank of Richmond Economic Quar-*
826 *terly*, 1993.
- 827 – **and Robert G. King**, “The incredible Volcker disinflation,” *Journal of*
828 *Monetary Economics*, July 2005, 52 (5), 981–1015.
- 829 **Gordon, Robert**, “The Phillips Curve is Alive and Well: Inflation and the
830 NAIRU During the Slow Recovery,” Technical Report aug 2013.
- 831 **Hamilton, James D.**, “A New Approach to the Economic Analysis of Non-
832 stationary Time Series and the Business Cycle,” *Econometrica*, 1989, 57 (2),
833 357–384.
- 834 **Ireland, Peter N.**, “Technology Shocks in the New Keynesian Model,” *Re-*
835 *view of Economics and Statistics*, November 2004, 86 (4), 923–936.
- 836 **Khan, Aubhik, Robert G. King, and Alexander L. Wolman**, “Optimal
837 Monetary Policy,” *Review of Economic Studies*, October 2003, 70 (4), 825–
838 860.
- 839 **King, Mervyn**, “Credibility and Monetary Policy: Theory and Evidence,”
840 *Bank of England Quarterly Bulletin*, March 2005.
- 841 **King, Robert G., Yang K. Lu, and Ernesto S. Pastén**, “Managing
842 Expectations,” *Journal of Money, Credit and Banking*, December 2008, 40
843 (8), 1625–1666.
- 844 **Kydland, Finn E. and Edward C. Prescott**, “Rules Rather than Discre-
845 tion: The Inconsistency of Optimal Plans,” *Journal of Political Economy*,
846 1977, 85 (3), 473–491.

- 847 — and —, “Dynamic optimal taxation, rational expectations and optimal
848 control,” *Journal of Economic Dynamics and Control*, January 1980, 2, 79–
849 91.
- 850 **Levin, Andrew and John B. Taylor**, “Falling Behind the Curve: A Positive
851 Analysis of Stop-Start Monetary Policies and the Great Inflation,” in “The
852 Great Inflation: The Rebirth of Modern Central Banking,” University of
853 Chicago Press, June 2013, pp. 217–244.
- 854 **Lu, Yang K.**, “Optimal policy with credibility concerns,” *Journal of Eco-*
855 *nomic Theory*, September 2013, 148 (5), 2007–2032.
- 856 —, **Robert G. King, and Ernesto Pasten**, “Optimal reputation building
857 in the New Keynesian model,” *Journal of Monetary Economics*, December
858 2016, 84, 233–249.
- 859 **Mailath, George J. and Larry Samuelson**, *Repeated Games and Reputa-*
860 *tions: Long-run Relationships*, Oxford University Press, July 2006.
- 861 **Marcet, Albert and Ramon Marimon**, “Recursive Contracts,” *Econo-*
862 *metrica*, 2019, 87 (5), 1589–1631.
- 863 **Matthes, Christian**, “Figuring Out the Fed-Beliefs about Policymakers and
864 Gains from Transparency,” *Journal of Money, Credit and Banking*, January
865 2015, 47 (1), 1–29.
- 866 **Melosi, Leonardo**, “Signalling Effects of Monetary Policy,” *The Review of*
867 *Economic Studies*, sep 2016, p. rdw050.
- 868 **Meltzer, Allan H.**, *A history of the Federal Reserve. vol. 2, book 2: 1970 -*
869 *1986*, paperback ed ed., Chicago: Univ. of Chicago Press, 2014.
- 870 **Mertens, Elmar and James M. Nason**, “Inflation and professional forecast
871 dynamics: An evaluation of stickiness, persistence, and volatility,” *Quanti-*
872 *tative Economics*, November 2020, 11 (4), 1485–1520.
- 873 **Mishkin, Frederic S.**, “Central Bank Commitment and Communication,”
874 April 2008.
- 875 — and **Klaus Schmidt-Hebbel**, “Does Inflation Targeting Make a Differ-
876 ence?,” NBER Working Papers 12876 January 2007.
- 877 **Orphanides, Athanasios**, “Comment on: “The incredible Volcker disinfla-
878 tion”,” *Journal of Monetary Economics*, July 2005, 52 (5), 1017–1023.
- 879 — and **John C. Williams**, “Imperfect Knowledge, Inflation Expectations,
880 and Monetary Policy,” in “The Inflation-Targeting Debate,” University of
881 Chicago Press, 2005, pp. 201–245.

- 882 — and —, “Monetary Policy Mistakes and the Evolution of Inflation Expecta-
883 tions,” in “The Great Inflation: The Rebirth of Modern Central Banking,”
884 University of Chicago Press, June 2013.
- 885 **Parkin, Michael**, “Inflation in North America,” in “Price Stabilization in
886 the 1990s,” Palgrave Macmillan UK, 1993, pp. 47–93.
- 887 **Phelan, Christopher and Ennio Stacchetti**, “Sequential Equilibria in a
888 Ramsey Tax Model,” *Econometrica*, 2001, 69 (6), 1491–1518.
- 889 **Primiceri, Giorgio E**, “Why Inflation Rose and Fell: Policy-Makers’s Beliefs
890 and U. S. Postwar Stabilization Policy,” *Quarterly Journal of Economics*,
891 aug 2006, 121 (3), 867–901.
- 892 **Roger, Scott and Mark R. Stone**, “On Target? the International Expe-
893 rience with Achieving Inflation Targets,” *IMF Working Paper No. 05/163*,
894 2005.
- 895 **Sargent, Thomas J.**, “The Ends of Four Big Inflations,” in “Inflation:
896 Causes and Consequences,” Univ. of Chicago Press, 1982, pp. 41–98.
- 897 —, *The conquest of American inflation*, 1st print ed., Princeton, NJ: Princeton
898 University Press, 1999.
- 899 — and **Ulf Soderstrom**, “The conquest of American inflation: A summary,”
900 *Sveriges Riksbank Economic Review*, 2000, 3, 12–45.
- 901 **Schaumburg, Ernst and Andrea Tambalotti**, “An investigation of the
902 gains from commitment in monetary policy,” *Journal of Monetary Eco-*
903 *nomics*, 2007, 54 (2), 302–324.
- 904 **Shapiro, Adam and Daniel J. Wilson**, “The Evolution of the FOMC’s
905 Explicit Inflation Target,” *FRBSF Economic Letter*, April 2019.
- 906 **Smets, Frank and Raf Wouters**, “An Estimated Dynamic Stochastic Gen-
907 eral Equilibrium Model of the Euro Area,” *Journal of the European Eco-*
908 *nomics Association*, September 2003, 1 (5), 1123–1175.
- 909 **Stock, James H. and Mark W. Watson**, “Why Has U.S. Inflation Become
910 Harder to Forecast?,” *Journal of Money, Credit and Banking*, February
911 2007, 39 (s1), 3–33.
- 912 **Watson, Mark W.**, “Inflation Persistence, the NAIRU, and the Great Re-
913 cession,” *American Economic Review*, May 2014, 104 (5), 31–36.
- 914 **Woodford, Michael**, “Macroeconomic Analysis Without the Rational Ex-
915 pectations Hypothesis,” *Annual Review of Economics*, August 2013, 5 (1),
916 303–346.

918 Appendices

919 **A Recursive optimal policy design**

920 The optimal policy problem for the committed type at the start of its tenure involves forward-
 921 looking constraints, which must be transformed to yield a recursive specification. Conceptu-
 922 ally, this involves casting Lagrangian components in recursive form, relying on (i) application
 923 of the law of iterated expectation and (ii) appropriate rearrangement of expected discounted
 924 sums. In the current model, the transformation to recursive form must also take into account
 925 that the committed policymaker and the private sector have different discount factors and
 926 probability beliefs, so that the law of iterated expectation must be applied carefully.

927 This appendix’s derivation of the recursive program in Proposition 1 incorporates three
 928 structural features described in section 2 of the text: (1) informational subperiods; (2)
 929 different information sets for the committed policymaker and the private sector; and (3)
 930 private sector learning. It also generalizes the section 2 framework so that it can be used
 931 with constant reputation or a mechanical alternative type. Various elements from the main
 932 text are repeated, so that the appendix may be read separately.

933 The detailed derivation of the recursive form is a slow-moving proof, designed for readers
 934 with various degrees of prior exposure to recursive optimal policy design. A key new feature
 935 relative to other macro applications is a “change of measure” in the expectations constraint
 936 on the committed policymaker, which arises because private agents understand that inflation
 937 may come from the decisions of an optimizing alternative type.¹

938 As we develop the optimal policy for the committed type, we assume that the committed
 939 type takes as given a function governing private agents’ expected inflation in the event of its
 940 replacement, which may depend on events during its tenure and, in particular, on its terminal
 941 reputation. But in the background, there is an equilibrium requirement that private agents
 942 form rational beliefs about inflation in the event of a replacement next period. We discuss
 943 imposing this requirement at the end of this appendix.

944 **A.1 Intended and actual inflation**

945 At each date, the policymaker chooses intended inflation, denoted as a for the committed
 946 type ($\tau = 1$) and α for the alternative type ($\tau = 2$). Intended inflation is not observed by the

¹This feature will play an even more important role in future research that makes the alternative type care more about the future than in the current case of a myopic alternative.

947 private sector. Actual inflation is randomly distributed around this intention, with density
 948 $g(\pi|a)$ if there is a committed type and $g(\pi|\alpha)$ if there is an alternative type. We assume

$$949 \quad a = \int \pi g(\pi|a, \tau = 1) d\pi$$

$$950 \quad \alpha = \int \pi g(\pi|\alpha, \tau = 2) d\pi$$

951 Implementation errors are $\varepsilon_1 = \pi - a$ and $\varepsilon_2 = \pi - \alpha$ for the two types. While we allow
 952 for different continuous distributions on the same range of inflation outcomes, we do not
 953 separately include type τ as an argument to avoid notation clutter in the balance of this
 954 appendix (i.e., we write $g(\pi|a)$ and $g(\pi|\alpha)$).

955 **A.2 Measures of history**

956 We use period t as the time index within a regime, so period 0 is the date of last regime
 957 change. The committed type begins with a reputation, ρ_0 , known to private agents.

Private agents at the end of period t know the entire history of inflation (π), output (x),
 and inflation shocks (ς) since period 0 (the last regime change date). After the next period
 starts, the ς shock is realized. The policymaker's intended inflation (a or α) is conditioned
 on this information, as is the expectations shifter in the output-inflation trade-off, e . We
 write the information history as

$$h_t = [\varsigma_t, \{\varsigma_{t-s}\}_{s=1}^t, \{\pi_{t-s}\}_{s=1}^t]$$

After the policymaker chooses his intended inflation, actual inflation and output are real-
 ized. Other variables, notably private agents' updated belief about policymaker type, are
 conditioned on this extended information,

$$h_t^+ = [\pi_t, h_t].$$

Note that

$$h_{t+1} = [\varsigma_{t+1}, h_t^+] = [\varsigma_{t+1}, \pi_t, h_t]$$

958 **A word on notation:** In the Public Perfect Bayesian Equilibrium of our dynamic game,
 959 variables depend just on the relevant history (e.g., $a(h_t)$) and not separately on the date
 960 (e.g., $a_t(h_t)$). To further streamline some formulas, we will sometimes condense variables
 961 even further, writing $a(h_t)$ as a_t .

962 **A.3 Beliefs about current inflation**

963 Although private agents do not know the type of policymaker that is in place, at the start
 964 of period t , they have a prior belief ρ_t that there is a committed type which will choose a_t
 965 and a complementary prior belief $1 - \rho_t$ that there is an alternative type which will choose
 966 α_t . Accordingly, their rational likelihood of the outcome π_t is

967 (A22)
$$g(\pi_t|a_t)\rho_t + g(\pi_t|\alpha_t)(1 - \rho_t)$$

968 **A.4 Beliefs about policymaker type**

969 On observing inflation within a regime, private agents use Bayes' law to update their condi-
 970 tional probability that the current policymaker is the committed type

971 (A23)
$$\begin{aligned} \rho(h_t^+) &= \frac{g(\pi_t|a(h_t))\rho(h_t)}{g(\pi_t|a(h_t))\rho(h_t) + g(\pi_t|\alpha(h_t))(1 - \rho(h_t))} \\ &\equiv b(\pi_t, a(h_t), \alpha(h_t), \rho(h_t)) \end{aligned}$$

972

973 where the b function is a convenient short-hand and $h_t^+ = [\pi_t, h_t]$. As there is no information
 974 about type revealed by ς_{t+1} , $\rho(h_{t+1}) = \rho(h_t^+)$. This updating may be written

975 (A24)
$$\rho(h_t^+) = \frac{\rho(h_t)}{\rho(h_t) + \lambda(\pi_t, h_t)(1 - \rho(h_t))}$$

976 using the likelihood ratio $\lambda(\pi_t, h_t) \equiv \frac{g(\pi_t|\alpha(h_t))}{g(\pi_t|a(h_t))}$.

977 **A.5 Constructing expected inflation**

978 We now construct the private sector's expected inflation, $E\pi_{t+1}$, working backwards from
 979 the start of next period to the start of this period. We take into account that there will be
 980 a regime change ($n_{t+1} = 1$) with probability q and won't ($n_{t+1} = 0$) with probability $1 - q$.

If the committed type is known to be in place, with decision rule $a([\varsigma_{t+1}, h_t^+])$, then

$$E(\pi_{t+1}|h_{t+1}, \tau_{t+1} = 1) = a([\varsigma_{t+1}, h_t^+])$$

since intended inflation is the mean of realized inflation. Similarly,

$$E(\pi_{t+1}|h_{t+1}, \tau_{t+1} = 2) = \alpha([\varsigma_{t+1}, h_t^+])$$

981 Since the private sector will not know the type of policymaker in place at the start of next

982 period, expected inflation will be

$$983 \quad (A25) \quad E(\pi_{t+1}|h_{t+1}, n_{t+1} = 0) = \rho(h_{t+1})a(h_{t+1}) + (1 - \rho(h_{t+1}))\alpha(h_{t+1})$$

984 if there isn't a regime change. Without taking a stand on the details of reputation inheritance,
985 we simply define

$$986 \quad (A26) \quad E(\pi_{t+1}|h_{t+1}, n_{t+1} = 1) = z(h_{t+1})$$

987 as the private sector's expectation of inflation conditional on a replacement.

Stepping back now to period t , expected inflation conditional on h_t is

$$(A27) \quad E(\pi_{t+1}|h_t) = \rho(h_t) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [(1 - q) a(h_{t+1}) + qz(h_{t+1})] g(\pi_t|a(h_t)) d\pi_t \\ + (1 - \rho(h_t)) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [(1 - q) \alpha(h_{t+1}) + qz(h_{t+1})] g(\pi_t|\alpha(h_t)) d\pi_t$$

988 There may appear to be a conflict between this expression and (A25) that contains reputation
989 at $t+1$. But there is not. Weighting (A25) and (A26) by $(1 - q)$ and q and then integrating
990 over the private sector's belief about inflation (A22) leads directly to it. The simplicity arises
991 because (A22) also occurs in the denominator of the Bayesian updating expression (A23).

992 A.6 Intertemporal objective

993 We assume that the policymaker's intertemporal objective involves discounting at $\beta_1(1 - q)$,
994 where β_1 is its structural discount factor and $(1 - q)$ reflects discounting due to replacement.

$$995 \quad U_t = \underline{u}(a_t, e_t, \varsigma_t, \tau = 1) + (\beta_1(1 - q))E_t^c U_{t+1}$$

996 where $\underline{u}(a, e, \varsigma, \tau = 1) \equiv \int u(\pi, x(\pi, e), \varsigma, \tau = 1) g(\pi|a) d\pi$ is the expected momentary ob-
997 jective with x replaced by $x(\pi, e) = (\pi - e - \varsigma) / \kappa$, and the conditional expectation operator
998 $E_t^c(\cdot)$ is using the committed type's probability $p(h_{t+j})$ of a specific history h_{t+j} when his
999 actions generate inflation.

1000 More specifically, at any date t given the history h_t , the intertemporal objective is

$$1001 \quad (A28) \quad U_t = \sum_{j=0}^{\infty} (\beta_1(1 - q))^j \sum_{h_{t+j}} \frac{p(h_{t+j})}{p(h_t)} \underline{u}(a(h_{t+j}), e(h_{t+j}), \varsigma(h_{t+j}), \tau = 1)$$

1002 Given $h_{t+j} = [\varsigma_{t+j}, \pi_{t+j-1}, h_{t+j-1}]$, the committed type's probability of a specific history is:

1003 (A29)
$$p(h_{t+j}) = \varphi(\varsigma_{t+j}; \varsigma_{t+j-1}) \times g(\pi_{t+j-1} | a(h_{t+j-1})) \times p(h_{t+j-1})$$

1004 That is, it combines the likelihood of inflation π given the committed type's decision, the
1005 likelihood of the shock ς and the probability of the previous history.²

1006 A.7 Rational expectations constraint

1007 To develop the desired recursive form, we construct the Lagrangian component using the
1008 committed type's probabilities as weights on the multipliers

1009 (A30)
$$\Psi_t = \sum_{j=0}^{\infty} (\beta_1(1-q))^j \sum_{h_{t+j}} \frac{p(h_{t+j})}{p(h_t)} \gamma(h_{t+j}) [e(h_{t+j}) - \beta E(\pi_{t+j+1} | h_{t+j})]$$

and then express it recursively. We detailed $E(\pi_{t+1} | h_t)$ in (A27), but the expression involved the probability of inflation under the alternative type. So, we undertake a “change of measure” and rewrite it as

(A31)
$$\begin{aligned} \rho(h_t) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [\beta(1-q)a(h_{t+1}) + \beta qz(h_{t+1})] g(\pi | a(h_t)) d\pi \\ + (1 - \rho(h_t)) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [\beta(1-q)\alpha(h_{t+1}) + \beta qz(h_{t+1})] \boldsymbol{\lambda}(\mathbf{h}_{t+1}) g(\pi | a(h_t)) d\pi \end{aligned}$$

1010 where $\lambda(h_{t+1})$ is the likelihood ratio discussed above in the context of Bayesian updating.

1011 (A32)
$$\frac{g(\pi_t | \alpha(h_t))}{g(\pi_t | a(h_t))} = \lambda(h_t^+) = \lambda(h_{t+1})$$

1012 As the notations emphasize, this is a random variable from the standpoint of h_t but it is
1013 known as of $h_t^+ = [\pi_t, h_t]$ and $h_{t+1} = [\varsigma_{t+1}, h_t^+]$.

1014 We now return to (A30) and replace $E(\pi_{t+1} | h_t)$ with the expression in (A31). Note that
1015 $a(h_{t+1})$, $\alpha(h_{t+1})\lambda(h_{t+1})$, and $z(h_{t+1})$ are multiplied by $\varphi(\varsigma_{t+1}; \varsigma_t)g(\pi | a(h_t))p(h_t)$ and by $\gamma(h_t)$,
1016 which is $p(h_{t+1})\gamma(h_t)$. So, just as in simpler models, it is possible to eliminate expectations at
1017 future dates, essentially by applying the law of iterated expectation. Adjusting for different

²We ask for the reader's patience in using a sum over histories to capture the joint effects of the possibly continuous distribution of π and the discrete Markov chain distribution for ς .

1018 discount factors, we can write (A30) as

$$1019 \quad (A33) \quad \Psi_t = E_t^c \left[\sum_{j=0}^{\infty} (\beta_1(1-q))^j \psi_{t+j} \right]$$

1020 with

$$1021 \quad (A34) \quad \psi_t = \gamma_t e_t - \frac{\beta}{\beta_1(1-q)} \gamma_{t-1} \{ \rho_{t-1} [(1-q)a_t + qz_t] + (1 - \rho_{t-1}) \lambda_t [(1-q)\alpha_t + qz_t] \}$$

1022 This latter expression captures past commitments about current state-contingent decisions
 1023 as these were relevant to past expectations of inflation.³ Note that at the start of the regime,
 1024 when $t = 0$, $\gamma_{t-1} = 0$ by assumption. The initial condition on reputation specifies ρ_0 .

1025 A.8 The basic recursive specification

The preceding derivations suggest a recursive version of $U_t + \Psi_t$ with states $(\varsigma_t, \gamma_{t-1}, \rho_{t-1}, \lambda_t)$. For algebraic convenience, we define $\eta_t = \frac{\beta}{\beta_1(1-q)} \gamma_{t-1}$. Then, the recursive form as in [Marcet and Marimon \(2019\)](#) is

$$(A35) \quad W(\varsigma_t, \eta_t, \rho_{t-1}, \lambda_t) = \min_{\gamma} \max_{a, \alpha, e} \{ \underline{u}(a_t, e_t, \varsigma_t, \tau = 1) + \gamma_t e_t \\
 - \eta_t [\rho_{t-1} ((1-q)a_t + qz_t) + (1 - \rho_{t-1}) \lambda_t ((1-q)\alpha_t + qz_t)] \\
 + \beta_1(1-q) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) W(\varsigma_{t+1}, \eta_{t+1}, \rho_t, \lambda_{t+1}) g(\pi_t | a_t) d\pi_t \}$$

subject to the IC constraint

$$\alpha_t = Ae_t + B(\varsigma_t)$$

1026 with state dynamics (from the perspective of the committed type)

$$1027 \quad \eta_{t+1} = \frac{\beta}{\beta_1(1-q)} \gamma_t \text{ with } \gamma_{-1} = 0 \\
 1028 \quad \rho_t = \frac{\rho_{t-1}}{\rho_{t-1} + (1 - \rho_{t-1}) \lambda_t} \text{ given } \rho_0 \\
 1029 \quad \lambda_{t+1} = \lambda(\pi_t, a_t, \alpha_t) \text{ with probability } g(\pi_t | a_t)$$

1030 Defining $S_t = [\varsigma_t, \eta_t, \rho_{t-1}, \lambda_t]$, this program delivers optimal choices $a^*(S)$, $\alpha^*(S)$, $e^*(S)$,
 1031 $\gamma^*(S)$ with optimal state evolution induced by these decision rules. As is standard in recursive

³Our short hand notation replaces $\lambda(h_t)$ with λ_t . Given (A32), the likelihood ratio λ_t is predetermined in period t by actions and inflation outcome in period $t - 1$.

1032 systems, these rules also imply a value of the objective, $U^*(S)$.

1033 A.9 State space reduction

1034 For computational and analytical benefits, it is desirable to reduce the state space. We now
 1035 show how to eliminate the likelihood ratio (λ) from the state vector so that we only need
 1036 three state variables instead of four. Start by rewriting (21) as

$$1037 \quad (A36) \quad \psi_t = \gamma_t e_t - \frac{\beta}{\beta_1(1-q)} \gamma_{t-1} \rho_{t-1} \{[(1-q)a_t + qz_t] + \frac{(1-\rho_{t-1})\lambda_t}{\rho_{t-1}} [(1-q)\alpha_t + qz_t]\}$$

1038 Then, note that $\rho_t = \frac{\rho_{t-1}}{\rho_{t-1} + (1-\rho_{t-1})\lambda_t}$ implies that $\frac{(1-\rho_{t-1})\lambda_t}{\rho_{t-1}} = \frac{1-\rho_t}{\rho_t}$ so that Bayes' rule can
 1039 be used to eliminate λ_t . Substitution of this expression into that above yields

$$1040 \quad (A37) \quad \psi_t = \gamma_t e_t - \frac{\beta}{\beta_1(1-q)} \gamma_{t-1} \rho_{t-1} \{[(1-q)a_t + qz_t] + \frac{(1-\rho_t)}{\rho_t} [(1-q)\alpha_t + qz_t]\}$$

1041 which indicates that the states $(\varsigma_t, \eta_t, \rho_{t-1}, \lambda_t)$ can be reduced to $\varsigma_t, \mu_t = \frac{\beta}{\beta_1(1-q)} \gamma_{t-1} \rho_{t-1}$ and
 1042 ρ_t with the following transition rules for the endogenous states given ρ_0 :

$$1043 \quad (A38) \quad \mu_{t+1} = \frac{\beta}{\beta_1(1-q)} \gamma_t \rho_t \text{ with } \mu_0 = 0$$

$$1044 \quad (A39) \quad \rho_{t+1} = b(\pi_t, a_t, \alpha_t, \rho_t) \text{ with probability } g(\pi_t | a_t)$$

1045 A.10 Extended recursive program

1046 The recursive optimization (A35) can now be written with only three state variables. While
 1047 doing so, we extend the program to make it easy to shut down each of the two key mecha-
 1048 nisms: endogenous reputation and optimizing behavior by the alternative type.

$$1049 \quad (A40) \quad W(\varsigma_t, \rho_t^s, \mu_t) = \min_{\gamma} \max_{a, \alpha, e} \{u(a_t, e_t, \varsigma_t, \tau = 1) + \gamma_t e_t + \mu_t \omega_t \\ 1050 \quad \quad \quad + \beta_1(1-q) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) W(\varsigma_{t+1}, \rho_{t+1}^s, \mu_{t+1}) g(\pi; a_t) d\pi\}$$

where

$$\omega_t = -\{[(1-q)a_t + qz_t] + \frac{(1-\rho_t^s)}{\rho_t^s} [(1-q)\alpha_t + qz_t]\}$$

subject to the IC constraint

$$\alpha_t = \begin{cases} Ae_t + B(\varsigma_t) & \text{if optimizing alternative type} \\ \underline{\alpha}(\varsigma_t) & \text{if mechanical alternative type} \end{cases}$$

1051 with state dynamics allowing exogenous reputation ($y=0$) or endogenous reputation ($y=1$)

$$\begin{aligned}
1052 \quad \mu_{t+1} &= \frac{\beta}{\beta_1(1-q)} \gamma_t \rho_t \\
1053 \quad \rho_{t+1} &= y \rho_{t+1}^s + (1-y) \rho \\
1054 \quad \rho_{t+1}^s &= b(\pi_t, a_t, \alpha_t, \rho_t)
\end{aligned}$$

1055 The recursive program here is written in a general form that allows (i) optimizing or
1056 mechanical alternative type and (ii) endogenous or exogenous reputation. The program
1057 in Proposition 1 of the main text is a special form of (A40) where there is an optimizing
1058 alternative type and endogenous reputation. Hence, in that setting, there is no need to
1059 distinguish ρ^s from ρ .

1060 A.11 The Fixed Point Requirement

1061 At the start of a period, there is a reputation ρ_t of a policymaker that would continue its
1062 tenure. If there is a replacement, then we assume that this reputation is partly inherited by
1063 a new policymaker whose date clock is set to zero, which we write as $\rho_0 = l(\rho_{t+1})$.

1064 Since we now know that optimal policies take the form $a^*(\varsigma, \rho, \mu)$ and $\alpha^*(\varsigma, \rho, \mu)$ given a
1065 particular function $z(\varsigma, \rho)$, rational expectations across regimes requires that the expected
1066 inflation conditional on a replacement $z(\varsigma, \rho)$ must satisfy the following fixed point:

$$1067 \quad (A41) \quad z^*(\varsigma, \rho) = \rho_0 a^*(\varsigma, \rho_0, 0; z^*(\varsigma, \rho)) + (1 - \rho_0) \alpha^*(\varsigma, \rho_0, 0; z^*(\varsigma, \rho))$$

1068 where $\rho_0 = l(\rho)$ is the new policymaker's initial reputation as described above and we impose
1069 $\mu = 0$ as appropriate at the start of a regime.

1070 B Consolidation

1071 This appendix explains how to simplify the recursive program in Proposition 1 to the one in
1072 Proposition 2, via the implications of private sector's rational expectation constraint.

1073 B.1 Relationship between U and W

1074 If $W(\cdot)$ in (A40) is differentiable, there are two notable implications of this structure.

The **envelope theorem implication** for μ is

$$W_\mu(\varsigma_t, \rho_t^s, \mu_t) = -\{[(1-q)a_t + qz_t] + \frac{(1-\rho_t^s)}{\rho_t^s} [(1-q)\alpha_t + qz_t]\}$$

1075 The first order necessary condition for γ_t is

$$\begin{aligned}
1076 \quad 0 &= e_t - \beta_1(1-q) \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) \int W_\mu(\varsigma_{t+1}, \rho_{t+1}, \mu_{t+1}) \frac{\partial \mu_{t+1}}{\partial \gamma_t} g(\pi_t | a_t) d\pi_t \\
1077 \quad &= e_t - \beta \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) \int W_\mu(\varsigma_{t+1}, \rho_{t+1}, \mu_{t+1}) \rho_t g(\pi_t | a_t) d\pi_t
\end{aligned}$$

1078 where the state evolution equation (A38) implies $\partial \mu_{t+1} / \partial \gamma_t = \rho_t \beta / (\beta_1(1-q))$.

When combined with an updated version of the envelope theorem implication, this FOC recovers the private sector's rational expectation constraint as in (A31):

$$e_t = \beta \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) \left[[(1-q)a_{t+1} + qz_{t+1}] + \frac{(1-\rho_{t+1}^s)}{\rho_{t+1}^s} [(1-q)\alpha_{t+1} + qz_{t+1}] \right] \rho_t g(\pi_t | a_t) d\pi_t$$

where

$$\frac{1 - \rho_{t+1}^s}{\rho_{t+1}^s} = \frac{(1 - \rho_t) \lambda_{t+1}}{\rho_t}.$$

Hence, in equilibrium where the rational expectation constraint must hold, we obtain the following relationship between the value function $W(\cdot)$ and the optimized objective $U^*(\cdot)$:

$$\begin{aligned}
\text{(B1)} \quad W(\varsigma_t, \rho_t^s, \mu_t) - \mu_t \omega_t^* &= U^*(\varsigma_t, \rho_t^s, \mu_t) \\
&= \underline{u}(a_t^*, e_t^*, \varsigma_t, \tau = 1) + \dots \\
&\quad + \beta_1(1-q) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) U^*(\varsigma_{t+1}, \rho_{t+1}^s, \mu_{t+1}) g(\pi_t | a_t) d\pi_t
\end{aligned}$$

1079 where $\omega_t^* = -\{[(1-q)a_t^* + qz_t^*] + \frac{(1-\rho_t^s)}{\rho_t^s} [(1-q)\alpha_t^* + qz_t^*]\}$.

1080 B.2 Further consolidation

1081 We now show that imposing the rational expectation constraint (A31) on the choice of e_t
1082 implies Lemma 1, which allows us to further reduce the recursive program in Proposition 1
1083 to the one in Proposition 2. The key idea is that only the policy difference $\delta = a - \alpha$ matters
1084 rather than the levels of a and α .

Recall that (A31) comes from (A27) before undertaking a “change of measure”. So the

original form of the rational expectation constraint on e_t is:

$$(B2) \quad e_t = \beta \rho_t \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [(1-q) a_{t+1} + q z_{t+1}] g(\pi_t | a_t) d\pi_t \\ + \beta (1 - \rho_t) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) [(1-q) \alpha_{t+1} + q z_{t+1}] g(\pi_t | \alpha_t) d\pi_t$$

1085 with a_{t+1} , α_{t+1} , and z_{t+1} determined by the three states $(\varsigma_{t+1}, \rho_{t+1}, \mu_{t+1})$ through the equi-
1086 librium strategies: $a^*(\cdot)$, $\alpha^*(\cdot)$, and $z^*(\cdot)$.

1087 Recall $\rho_{t+1} = b(\pi_t, a_t, \alpha_t, \rho_t)$ from (A39) and $b(\cdot)$ is the Bayes' learning rule specified in
1088 (A23). The inflation distribution is $\pi = a + \varepsilon_1$ under the committed type and $\pi = \alpha + \varepsilon_2$
1089 under the opportunistic type, with ε_1 and ε_2 being zero mean random variables. We can
1090 therefore rewrite the Bayes' learning rule (A23) as

$$(B3) \quad \rho_{t+1} = \frac{\phi_1(\pi_t - a_t) \rho_t}{\phi_1(\pi_t - a_t) \rho_t + \phi_2(\pi_t - \alpha_t) (1 - \rho_t)} \\ \equiv b(\pi_t - a_t, \pi_t - \alpha_t, \rho_t)$$

1093 where $g(\pi | a) = \phi_1(\pi - a)$ and $g(\pi | \alpha) = \phi_2(\pi - \alpha)$, and the b function is a version of our
1094 general convenient short-hand which is identified by its three argument nature.

1095 Then, in terms of the policy difference $\delta = a - \alpha$, future reputation is

$$(B4) \quad \rho' = b(\varepsilon_1, \varepsilon_1 + \delta, \rho) \text{ conditional on } \tau = 1$$

$$(B5) \quad \rho' = b(\varepsilon_2 - \delta, \varepsilon_2, \rho) \text{ conditional on } \tau = 2$$

1098 Replacing $g(\pi | a)$ and $g(\pi | \alpha)$ in (B2) with $\phi_1(\pi - a)$ and $\phi_2(\pi - \alpha)$, ρ_{t+1} with (B4) and
1099 (B5), and realizing choosing γ_t is equivalent to choosing μ_{t+1} due to $\mu_{t+1} = \frac{\beta}{\beta_1(1-q)} \gamma_t \rho_t$, we
1100 obtain Lemma 1 with the added details as follows:

LEMMA 1. Given (ς, ρ) , and that future policymakers follow the equilibrium strategies $a^*(\varsigma', \rho', \mu')$, $\alpha^*(\varsigma', \rho', \mu')$ and $z^*(\varsigma', \rho')$, rationally expected inflation is uniquely determined by the contemporaneous policy difference $\delta = a - \alpha$, and the future pseudo-state variable μ' .

$$e = e(\delta, \mu'; \varsigma, \rho) = \beta\rho \int \widehat{M}_1(\varsigma, b(\varepsilon_1, \varepsilon_1 + \delta, \rho), \mu') \phi_1(\varepsilon_1) d\varepsilon_1 + \beta(1 - \rho) \int \widehat{M}_2(\varsigma, b(\varepsilon_2 - \delta, \varepsilon_2, \rho), \mu') \phi_2(\varepsilon_2) d\varepsilon_2;$$

where $\phi_1(\cdot)$ and $\phi_2(\cdot)$ denote the density functions of ε_1 and ε_2 ;

$$\begin{aligned} \widehat{M}_1(\varsigma, \rho', \mu') & : = \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) [(1 - q) a^*(\varsigma', \rho', \mu') + qz^*(\varsigma', \rho')]; \\ \widehat{M}_2(\varsigma, \rho', \mu') & : = \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) [(1 - q) \alpha^*(\varsigma', \rho', \mu') + qz^*(\varsigma', \rho')]; \end{aligned}$$

Lemma 1 enables us to simplify the recursive program in Proposition 1, moving from choosing (γ, a, α, e) to merely choosing (δ, μ') . More specifically, once $e(\delta, \mu'; \varsigma, \rho)$ is chosen via the choices of (δ, μ') , we can obtain α from $Ae + B(\varsigma)$, and a from $\alpha + \delta$.

Furthermore, the relationship between U and W specified in (B1) implies that the objective of the recursive optimization can be reduced to

$$\underline{u}(a, e, \varsigma, \tau = 1) + \mu\omega(a, \alpha) + \beta_1(1 - q) \int \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) U^*(\varsigma', \rho', \mu') g(\pi|a) d\pi$$

where $U^*(\varsigma, \rho, \mu) = W(\varsigma, \rho, \mu) - \mu\omega(a^*, \alpha^*)$.

Replacing (e, α, a) in $\underline{u}(\cdot)$ and $\omega(\cdot)$ with $e(\delta, \mu'; \varsigma, \rho)$, $Ae + B(\varsigma)$, and $\alpha + \delta$ makes u and ω only depend on (δ, μ') :

$$(B6) \quad \underline{\underline{u}}(\delta, \mu') := \underline{u}(Ae + B(\varsigma), e, \varsigma, \tau = 1)$$

$$(B7) \quad \underline{\underline{\omega}}(\delta, \mu') := -\frac{1}{\rho} [(1 - q)(Ae + B(\varsigma)) + qz^*(\varsigma, \rho)] - (1 - q)\delta$$

where $e = e(\delta, \mu'; \varsigma, \rho)$. Replacing ρ' in $U^*(\cdot)$ with (B4) and $g(\pi|a)$ with $\phi_1(\varepsilon_1)$, we then arrive at the recursive program in Proposition 2.

C Forecasting Functions and Matching the SPF

C.1 SPF Data

This project is our first use of Survey of Professional Forecasters data. Many researchers employ the summary data files from the Federal Reserve Bank of Philadelphia, particularly the “annualized percent change of median responses” file, available for the GDP deflator at <https://www.philadelphiafed.org/surveys-and-data/pgdp>. This file includes an inflation “nowcast” and forecasts at the 1,2,3, and 4 quarter horizons.

In the middle of each quarter, each survey participant submits a forecast for the price level in that quarter and the next four. The FRBP first constructs a median price level for each horizon, say $P_{t+k|t}$ for $k=0,1,\dots,4$. It then constructs an annualized percentage growth rate using the formula $100 * ([P_{t+k|t}/P_{t+k-1|t}]^4 - 1)$.

The series used in our research differ in two ways. First, we compute annualized percent growth rates as $400 * \log(P_{t+k|t}/P_{t+k-1|t})$. Second, we start by calculating these growth rates for each forecaster at each date. We then take the median of these inflation rates.

Our procedure yields time series that are less prone to transitory outliers than the standard FRBP constructions. Each difference matters, i.e., (i) the median of the inflation rates is less prone than is the change in the median price level; and (ii) the continuously compounded inflation rate is less prone than is the FRBP inflation rate.

At some point, we plan to investigate these differences in more detail, as well as looking into the behavior of mean and trimmed mean inflation rates, but the time series employed seemed to us to be the best combination of conventional practice and attention to the underlying survey data. As our theory does not start with microfoundations, it is silent on the best manner to undertake such constructions.

C.2 Recursive forecasting in our theory

The SPF contains multiperiod forecasts of inflation. Real and nominal interest rates contain multiperiod forecasts of output and inflation. This appendix describes the calculation of such forecasts. We specialize the inflation distributions to

$$(C1) \quad \pi_t = a_t + \sigma \varepsilon_t \quad \text{and} \quad \pi_t = \alpha_t + \sigma \varepsilon_t$$

1136 with a density $\phi(\varepsilon)$ compatible with a zero mean and a unit standard deviation such as the
 1137 standard normal.⁴

1138 The information set is assumed to be the start of period information of the private sector,
 1139 $(\varsigma_t, \rho_t, \mu_t)$. Generally, our approach is applicable to forecasting any variable v_{t+k} which has
 1140 a functional solution

$$1141 \quad v(\varsigma_t, \rho_t, \mu_t)$$

1142 that is known to private agents and our specific applications are to inflation and output.

1143 C.2.1 Forecasting inflation

1144 Let us start with forecasting inflation k steps ahead, which we denote $f_{t+k|t}$.⁵ Private agents
 1145 know the intended inflation functions of the two policymakers:

$$1146 \quad a(\varsigma_t, \rho_t, \mu_t)$$

$$1147 \quad \alpha(\varsigma_t, \rho_t, \mu_t)$$

1148 Accordingly, given that implementation errors have mean zero, the private sector “nowcast”
 1149 of inflation is

$$1150 \quad f_{t|t} = f(\varsigma_t, \rho_t, \mu_t, 0) = \rho_t a(\varsigma_t, \rho_t, \mu_t) + (1 - \rho_t) \alpha(\varsigma_t, \rho_t, \mu_t)$$

1151 Utilizing the law of iterated expectation, today’s forecast of π_{t+j} is today’s forecast of
 1152 tomorrow’s forecast of π_{t+j} . We can compute multistep forecasts of inflation recursively
 1153 building up $f_{t+j|t}$ from $f_{t+j|t+1}$:

$$1154 \quad (\text{C2}) \quad f_{t+j|t} = f(\varsigma_t, \rho_t, \mu_t, j) = E_t(f_{t+j|t+1}) = E_t[f(\varsigma_{t+1}, \rho_{t+1}, \mu_{t+1}, j - 1)]$$

1155 The state variables ρ_{t+1} and μ_{t+1} evolve as follows.

1156 With probability $1 - q$ there is no regime change. The pseudo-state variable is evolves
 1157 according to:

$$1158 \quad \mu_{t+1} = \mu'^*(\varsigma_t, \rho_t, \mu_t).$$

⁴The recipe allows for type dependent parameters σ_1 and σ_2 but we use the common σ assumption for simplicity in this discussion.

⁵The model solution already contains a one-step ahead forecast for inflation as a function of the state, i.e. $f_{t+1|t} = f(\varsigma_t, \rho_t, \mu_t, 1) = e^*(\varsigma_t, \rho_t, \mu_t)/\beta$. Our concern here is longer-term inflation.

1159 The reputation state variable ρ_{t+1} evolves according to:

$$\begin{aligned}
 1160 \quad & \rho_{t+1} = b(a_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t) \quad \text{with prob } \rho_t \\
 1161 \quad & \rho_{t+1} = b(\alpha_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t) \quad \text{with prob } 1 - \rho_t
 \end{aligned}$$

1162 With probability q there is a regime change, in which case $\mu_{t+1} = 0$ and ρ_{t+1} evolves
 1163 according to an inheritance mechanism that relates the new policymaker's initial reputation
 1164 ρ_0 to what it would have been if there was no replacement, i.e., ρ_{t+1} , which we write as
 1165 $\rho_0 = l(\rho_{t+1})$.

1166 Then, we can determine

$$\begin{aligned}
 1167 \quad (C3) \quad & f_{t+j|t} = f(\varsigma_t, \rho_t, \mu_t, j) = \sum \varphi(\varsigma_{t+1}; \varsigma_t) \{ \\
 1168 \quad & (1 - q)\rho_t \int f[\varsigma_{t+1}, b(a_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t), \mu_{t+1}, j - 1] \phi(\varepsilon) d\varepsilon \\
 1169 \quad & + (1 - q)(1 - \rho_t) \int f[\varsigma_{t+1}, b(\alpha_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t), \mu_{t+1}, j - 1] \phi(\varepsilon) d\varepsilon \\
 1170 \quad & + q\rho_t \int f[\varsigma_{t+1}, l(b(a_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t)), 0, j - 1] \phi(\varepsilon) d\varepsilon \\
 1171 \quad & + q(1 - \rho_t) \int f[\varsigma_{t+1}, l(b(\alpha_t + \sigma\varepsilon_t, a_t, \alpha_t, \rho_t)), 0, j - 1] \phi(\varepsilon) d\varepsilon \}
 \end{aligned}$$

1172 C.2.2 Forecasting output

1173 We now turn to forecasting output, determined by

$$1174 \quad x_t = \frac{1}{\kappa} [\pi_t - \beta f(\varsigma_t, \rho_t, \mu_t, 1) - \varsigma_t]$$

1175 so that a “nowcast” of output is

$$1176 \quad \hat{x}_0(\varsigma_t, \rho_t, \mu_t) = \frac{1}{\kappa} [f(\varsigma_t, \rho_t, \mu_t, 0) - \beta f(\varsigma_t, \rho_t, \mu_t, 1) - \varsigma_t]$$

1177 Hence, we can use the same recipe for multistep forecasts:

$$1178 \quad \hat{x}_{j+1}(\varsigma_t, \rho_t, \mu_t) = E_t[\hat{x}_j(\varsigma_{t+1}, \rho_{t+1}, \mu_{t+1})]$$

1179 recursively building up \hat{x}_{j+1} from \hat{x}_j .

1180 C.3 Matching the SPF: motivation and mechanics

1181 From the standpoint of modern econometrics, our theory is a very simple one that is easily
 1182 rejected: conditional on regime change dates and the identification of policymaker type
 1183 within each regime: we have just two random inputs – price shocks ς_t and implementation
 1184 errors ε_t – that drive many observable macro time series. To review, there are three state
 1185 variables $s_t = [\varsigma_t, \rho_t, \mu_t]$, governed by a Markov process with a special form

$$\begin{aligned}
 1186 \quad \varsigma_t &= \nu\varsigma_{t-1} + \xi_t \\
 1187 \quad \rho_{t+1} &= b(\pi_t, a^*(s_t), \alpha^*(s_t), \rho_t) \\
 1188 \quad \mu_{t+1} &= \mu^*(\varsigma_t, \rho_t, \mu_t)
 \end{aligned}$$

1189 Many variables depend just on these states, including the policies a_t and α_t and, as we just
 1190 discussed, expectations at various horizons $f_{t+k|t}$. Others, including inflation π_t and real
 1191 activity x_t , also depend on ε_t .

1192 Our work in this paper is quantitative theory and, following early RBC analyses, we
 1193 fix model parameters and use a transparent strategy for extracting the unobserved states.
 1194 Then, with the states in hand, we calculate the historical behavior of observables.⁶ But
 1195 the literature has stressed that one of the difficulties with this RBC strategy is that the
 1196 technology state is measured by the Solow residual, which is based on observable variables
 1197 (output, capital, and labor) whose behavior is ultimately to be explored.

1198 C.3.1 The strategy for extracting states

We therefore develop a strategy for extracting state information that does not use the be-
 havior of the GDP deflator. It relies on the fact that our model provides a mapping between
 states and inflation expectations at various horizons:

$$f_{t+k|t} = f(\varsigma_t, \rho_t, \mu_t, k).$$

1199 Since the pseudo state μ_t is predetermined, we can solve for $\widehat{\varsigma}_t$ and $\widehat{\rho}_t$ from two elements of the
 1200 SPF term structure, if we identify model expectations at horizon k with the k -quarter-ahead
 1201 SPF inflation forecast.

1202 With the date t extracted states $\widehat{\varsigma}_t$ and $\widehat{\rho}_t$ and the predetermined state $\widehat{\mu}_t$ in hand, we

⁶Prescott (1986) constructs Solow residuals as productivity indicators and then calculates moment impli-
 cations for many variables of a model with calibrated parameters. Our work is closer to Plosser (1989), who
 uses the Solow residual time series and a basic calibrated model to construct time series of many variables,
 including consumption, investment and so on.

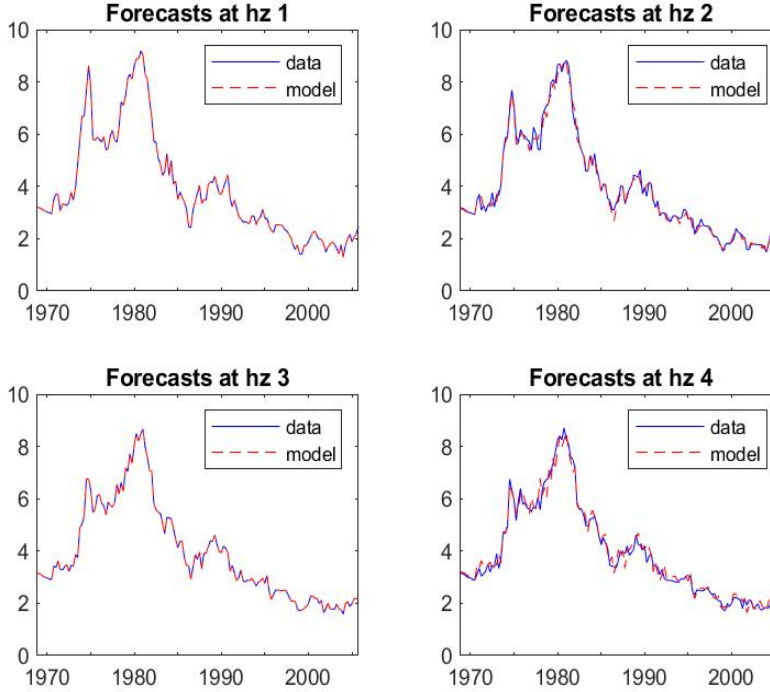


Figure 11: Model-implied and SPF forecasts of inflation

1203 can create $\hat{\mu}_{t+1}$ using the third transition rule, $\mu^*(\cdot)$, continuing recursively to calculate a
 1204 full history of states. It is true that we need an initial condition on μ , but that is supplied
 1205 by specifying a set of regime switch dates at which μ is set zero.

1206 In the main text, we use $\mu'^*(0, \hat{\rho}_t, \hat{\mu}_t)$ to determine $\hat{\mu}_{t+1}$ recursively, instead of $\mu'^*(\hat{\zeta}_t, \hat{\rho}_t, \hat{\mu}_t)$.
 1207 We do so because it is a natural way to preserve the mean-reverting property of ζ shock in
 1208 the extract $\hat{\zeta}_t$ series. We nonetheless redo our quantitative fitting exercise with a version
 1209 of state extraction using $\hat{\mu}_{t+1} = \mu'^*(\hat{\zeta}_t, \hat{\rho}_t, \hat{\mu}_t)$. The model's fitting to the U.S. inflation is
 1210 similar to that reported in the main text. The results are reported in Section C.4.

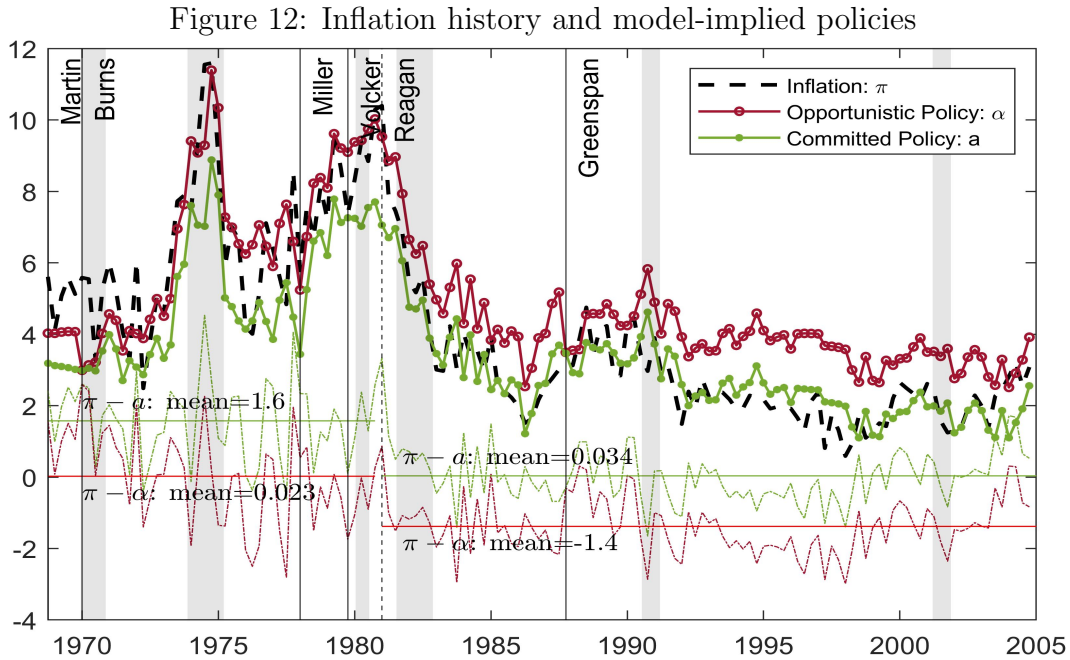
1211 C.3.2 Application and fitting performance

1212 As discussed in section 6.1 of the main text, we extract latent states by matching model-
 1213 implied inflation forecasts at horizons 1 and 3 with SPF one-quarter-ahead and three-quarter-
 1214 ahead forecasts. The left panels in Figure 11 shows our match is nearly perfect given that
 1215 we choose two state variables ζ and ρ each period to match two data points SPF1Q and
 1216 SPF3Q. Using the extracted states, we can also compute model-implied inflation forecasts
 1217 at horizons 2 and 4, and compare them with SPF two-quarter-ahead and four-quarter-ahead

1218 forecasts. The comparison is shown in the right panels of Figure 11. It is notable that
 1219 our model-implied forecasts lie almost entirely on top of the SPF data for both forecasting
 1220 horizons, which are not explicitly targeted. We view this figure as evidence in support of
 1221 our state extraction approach.

1222 C.4 Results without imposing mean-reverting on extracted $\hat{\zeta}$

1223 The results in the main text are based on using a decision rule $\hat{\mu}' = \mu'^*(0, \hat{\rho}, \hat{\mu})$ rather than
 1224 $\hat{\mu}' = \mu'^*(\hat{\zeta}, \hat{\rho}, \hat{\mu})$. That is, we do not allow the extracted shock to influence the dynamics of
 1225 the pseudo state variable. Figure 12 displays the results when we alternatively allow this
 influence. The main messages from the text are maintained.



Note: Model-implied policies are based on extracted states produced using $\hat{\mu}' = \mu'^*(\hat{\zeta}, \hat{\rho}, \hat{\mu})$ recursively.

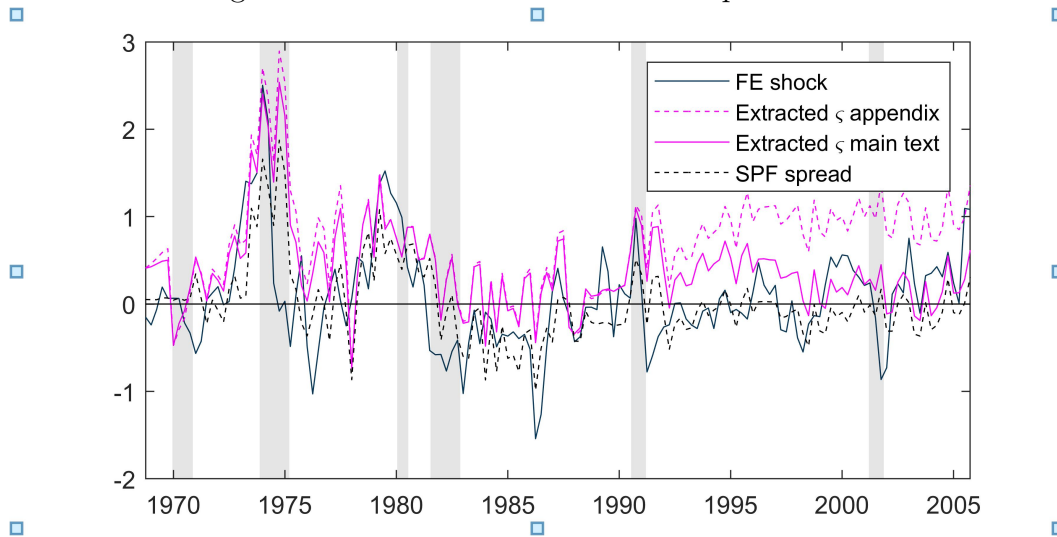
1226

1227 C.5 Shock Comparisons

1228 We have explored four indicators of the cost-push shock. First, there is a food and
 1229 energy shock constructed along the lines of [Watson \(2014\)](#). Second, there is the SPF spread.
 1230 Third, there is the extracted shock series from the main text. Fourth, there is the extracted
 1231 shock using the procedure that we just discussed. Figure 13 displays these alternative series.
 1232 Note first that all measures rise dramatically during the famous “oil price shock” of late 1973
 1233 and early 1974 and also during the late 1970s interval that preceded Volcker’s appointment.

1234 Note next that the extracted shocks and the SPF spread are more persistent during the
 1235 earlier episode. Contemporary sources, such as the January 1975 Economic Report of the
 1236 President prepared by Alan Greenspan and his CEA colleagues, point to other price shocks in
 1237 addition to oil during the preceding year. Econometric studies such as those of [R.J. Gordon](#)
 1238 [\(2013\)](#) and [Watson \(2014\)](#) estimate price shocks, including those from price decontrols in
 1239 the 1970s, of more lasting form. So, on this basis, we are led to prefer extracted shocks as
 1240 a parsimonious approach. Note further that the extracted shocks depart from each other
 1241 toward the end of the period, which is the motivation for us to adopt the extraction strategy
 1242 employed in the main text rather than that discussed in the prior section. Our extraction
 1243 procedure is a straightforward and transparent way to induce the extracted price shocks
 1244 to be mean-reverting, but does not explicitly impose the requirement that extracted price
 1245 shocks are stationary. More sophisticated methods, applicable to hidden Markov models
 such as ours, would impose that requirement.

Figure 13: Various indicators for cost-push shocks



Note: Comparing $\hat{\zeta}$ extracted using $\hat{\mu}' = \mu'^*(\hat{\zeta}, \hat{\rho}, \hat{\mu})$ to its counterpart in main text using $\hat{\mu}' = \mu'^*(0, \hat{\rho}, \hat{\mu})$, the two series behave similarly before 1990, but the former is higher than the latter after 1990. The FE shock is the “Food and Energy price shock,” constructed as the difference between the growth rate of the overall personal consumption deflator and its counterpart excluding food and energy. SPF spread is SPF1Q-SPF3Q.

1246