Managing Expectations*

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Abstract

The idea that monetary policy is principally about "managing expectations" has taken hold in central banks around the world. Discussions of expectations management by central bankers, academics and by financial market participants frequently also include the idea that central bank credibility is imperfect. We adapt a familiar macroeconomic model so as to discuss key concepts in the area of expectations management. Our work also exemplifies a model construction approach to analyzing the dynamics of announcements, actions and credibility that we think makes feasible a wide range of future investigations concerning the management of expectations.

Keywords: managing expectations, imperfect credibility, monetary policy

JEL codes: E3, E5, E6

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1 Introduction

It is increasingly standard for central bankers, financial market participants, and academic researchers to describe management of expectations as central to monetary policy.[1] The idea of "managing expectations" arises in many specific contexts including the rationale for particular policy actions, the optimal choice of the monetary instrument, the form of central bank behavior during leadership transitions, the desirable central bank response to frequent or unusual shocks, and the nature of monetary policy necessary for disinflation. Analysts who stress management of expectations frequently also suggest that credibility for low and stable inflation is imperfect, making expectations management subtle.[2]

It is therefore desirable to have macroeconomic models in which active management of expectations can be systematically examined. In this paper, we describe a strategy for constructing such models and we put it to work. Adapting a simple and familiar model, we begin by highlighting the relationship between policy announcements, beliefs, and policy actions that is at the heart of the management of expectations. Our view is that there are five features critical to the interplay between expectations management and credibility. We think that private agents do not know the underlying nature of the central bank and, in particular, whether it will take the actions necessary to produce low and stable inflation in the longer run: we define *long-term credibility* of the central bank as the private sector's likelihood that the central bank is of such a strong type. We think that private agents also are uncertain about whether near-term central bank actions will be those consistent with the central bank being of a strong type: we define the *short-term credibility* of the central bank as the private sector's likelihood that the central bank will take such actions. We think that short-term credibility is generally higher than long-term

[1] A recent financial market discussion sets the tone: "If monetary policy is managing expectations, as the current doctrine has it..." *Central Banking News*, April 2006. A former central banker, Alan Blinder uses the terminolgy in a more specific context: "Forward-looking information helps condition ("manage") expectations" *Making Monetary Policy and Talking* about it, a presentation at University of Chicago-Graduate School of Business in March 2007.

[2] Phillip Hildebrand, vice chairman of the SNB board has recently written that "Monetary policy could therefore be defined as the process of managing expectations about the future path of the monetary policy instrument and the probable effects of that path on the economy." He also stresses the issues that concern us in this paper: "Monetary policy works primarily through expectations. Transparency and credibility render monetary policy more effective. However, they are no substitutes for action. If a credible central bank uses words with the explicit aim of substituting them for action, it will risk losing credibility." (2006).

credibility because of *mimicking*, the possibility that the central bank will take the same near-term actions as a low-inflation central bank even it is of a weak type, i.e., even if low long-term inflation will not be the outcome under its optimal behavior.[3] We think that the *announced inflation plans* of a strong central bank will influence inflation expectations and that the central bank will use its plans to manage inflation expectations at a point in time as well as the evolution of its long-term credibility. Finally, we think that inflation plan announcements are subject to *imitation* by weak central banks, so that such plans are not perfectly credible.

To display how these core ideas about expectations management can be introduced into macroeconomic models and the consequences of doing so, we adapt a framework familiar from the work of Kydland and Prescott [1977] and Barro and Gordon [1983] on central bank behavior under commitment and discretion. This choice reflects the fact that the starting point for our research is work by Barro [1986], Backus and Driffill [1985a,b], and Cukierman and Leviatan [1991], which studied variants of the KP-BG model in which policymaker type was assumed not observable. This prior research considered two types of monetary authority – a strong type capable of commitment to low inflation and a weak type that might engage in discretionary behavior – and focused on the interplay between central bank behavior and private sector learning about policymaker type. In turn, these analyses drew heavily on prior game-theoretic work by Kreps and Wilson [1982], that provided a basic model of reputation acquisition under imperfect information about player type. In common with this prior work, we aim to produce models that have a unique Markovian equilibrium, so that unambiguous predictions can be made. However, to apply the Kreps and Wilson approach, prior research adopted behavioral specifications which precluded analysis of important aspects of the management of expectations in monetary policy. For Barro [1986] and Backus and Driffill [1985a,b], the strong monetary authority was essentially an automaton, interested in securing zero inflation, while their analysis focused on understanding the interplay between weak authority mimicking and private sector beliefs. For Cukierman and Leviatan [1991], the weak authority was an automaton, choosing inflation unrelated to the actions of the committed monetary authority and

[3] The idea that mimicking is important for the dynamics of credibility is stressed by Phelan [2006]. That framework is poorly suited to our research focus, because the strong (low tax) policymaker is an automaton. However, Phelan's focus on unobserved shifts in policymaker type – contrasting with our observed regime replacement resulting in an unknown new type policymaker – is a very interesting direction for future work.

the beliefs of private agents.[4]

We adopt an alternative modeling strategy that we detail in section 2 below. Our weak monetary authority is subject to privately observable random discount factor shocks, i.e., shifts in the urgency attached to the present as opposed to the future. Technically, this exogenous randomness serves as an alternative "convexification" device for modelling economies so that randomized strategies are not a feature of our analysis. However, more basically, we think that it captures the idea that time-varying, exogenous factors influence the likelihood that a weak decisionmaker will behave in a discretionary manner. This approach permits us to exemplify core channels of expectation management by a strong central bank in simple static models and quantify the effects of expectation management in more elaborate dynamic models. Overall, it allows us to bring the analysis of optimal policy with imperfect information and partial commitment closer to recent work on optimal policy design along Ramsey [1927] lines that assumes perfect information and full commitment.

Within a static variant of our framework, we begin in sections 3 and 4 by showing that the strong central bank will not choose zero inflation in a setting of imperfect credibility, an idea stressed by Cukierman and Leviatan [1991]. Instead, the strong central bank will choose to announce and carry out a positive rate of inflation, with the extent of this inflation bias depending inversely on the extent of its credibility. Further, in our framework, the strong central bank will take into account the effect of its plan on private sector beliefs about what a weak central bank would do, both in terms of the rate of inflation that it would select if it opts to behave in a discretionary manner (in section 3) and the likelihood that it would instead mimic the strong central bank's inflation action (in section 4).[5] We also use this static model to discuss the role of inflation plan announcements in section 5: like Cukierman and Leviatan [1991], we view these announcements as an essential element of central bank transparency which allows for a strong central bank to bring about its desired management of expectations as part of a signalling game with an imperfectly informed private sector. Although signalling games with costless imitation in announcements – such as that by

[4] In our view, the Kreps and Wilson approach has important drawbacks as a framework for the analysis of the interplay between expectation management and macroeconomic activity, since it involves randomized strategies by the weak policymaker and may not make interesting predictions about the evolution of a credibility state variable when decisionmaker horizons are driven to an infinite horizon limit.

[5] In the static model, we use a random fixed cost of deviation to stand-in for the random discount factor in the full model.

a weak central bank in our framework – have not been much studied, we describe the natural equilibrium selection approach of Mailath, Okuna-Fujiwara, and Postelwhaite [1993] and explain why it leads to the same outcome that we derive using a more direct and familiar rational expectations approach. We conclude that management of expectations is generally not possible without *some* signals and that inflation plans are the necessary signal in our setting.

We analyze the dynamics of credibility within a setting in which the central bank has a very long horizon in section 6. Central bank policymaker tenure is, however, random. Each period, replacement occurs with a fixed probability: private agents know whether there has been a replacement, if one occurs, but do not know the type of a new central banker. Accordingly, our initial focus is on how a strong central bank builds long-term credibility, efficiently investing in reputational capital. We find that a strong central bank delivers a path of inflation that declines toward zero through time, with expected inflation also declining toward the same level. Private agents recognize that there is a chance of a surprise inflation by a weak policy-maker. But, as time passes, it becomes less likely that a weak central bank is in place (long-term credibility increases) and the likelihood of mimicking stabilizes at a positive level. When a replacement event occur, the new central banker again invests in credibility, either to ultimately secure zero inflation or to burn it in a surprise inflation.[6]

A virtue of our framework is that it can be readily extended to consideration of various types of shocks, which we illustrate with an application of current policy interest. Temporary positive oil price shocks have sometimes been suggested as the reason for lasting changes in the inflation rate, an idea investigated by Ball [1995] within a model that also features learning about policymaker type. The idea is that such a shock can induce a weak central bank to behave in a discretionary manner, revealing its type and leading to a period of high inflation until a new central bank occurs. However, in his analysis, the strong type always chooses zero inflation, as in

[6] Our dynamic analysis yields results that are quite different from the earlier literature in its predictions about the dynamics of inflation after a new policymaker is introduced. In that work, the tenure of the central bank is a fixed length. Early in the tenure, incentives for mimicking are highest and these are eroded as time passes, with weak central banks being more likely to engage in discretionary behavior as time passes, as stressed by Backus and Driffill [1985a,b] and Barro [1986], but the likelihood of a weak central bank being present lower. When the strong policymaker can choose his inflation rate, as in Cukierman and Leviatan [1991], the implication may well be that actual and expected inflation rises with the time that an apparently strong central bank has been in office. This contrasts with our model, but occurs for an understandable reason: the increase of time-in-office reduces the time left in office in a model with a fixed tenure.

Barro [1986] and Backus and Driffill [1985a,b]. Our framework allows us to investigate how the strong policymaker's incentives for accommodation to energy shocks depend on the extent of his credibility and, in turn, how this response affects the nature of private sector learning about policymaker type. At low levels of credibility, the strong policymaker faces a large shift in expected inflation when: private agents know that a weak central bank could well be in place and that it may choose to inflate if it places a lot of weight on current circumstances. Therefore, to avoid a large fall in output, the optimizing strong central bank will also choose a higher inflation rate, partially accommodating the shift in expected inflation. However, at high levels of credibility, the strong policymaker will maintain a low inflation rate in the face of a large oil price shock. Thus, our framework potentially can explain both why oil price shocks would have led to sustained periods of high inflation in earlier times and why there has been a much smaller effect on inflation in recent times, at least as of this writing.

In section 8, we conclude and discuss desirable future lines of inquiry.

2 Commitment, discretion, plans and credibility

The original analysis of Kydland and Prescott [1997] focused the debate about "rules versus discretion" on the "time inconsistency of optimal plans" within economies in which private agents form expectations rationally. They stressed that the use of optimal control theory — with its implicit assumption that the policymaker could commit to an entire future path of policy — leads to time inconsistency of optimal plans if the policymaker actually operates in a discretionary setting, making decisions on a period-by-period basis.

Like earlier models in the literature, our analysis focuses on the idea that policymakers can be of a strong type or weak type, with our setup having the following particular features.[7] First, we view strong policymakers as formulating an inflation plan p at the start of each period within our discrete time model. This plan is assumed to be observable by private agents, but we consider alterations of this assumption below. Second, we view strong decisionmakers as always able to execute

[7]Barro [1986] and Backus and Driffill [1985a,b] make the strong policymaker only care about inflation. We follow Cukierman and Leviatan [1991] in endowing the strong policymaker with the ability to execute an announced inflation plan.

the plan, producing inflation $\pi = p$ at the end of the period. A weak decisionmaker is assumed to state the same plan as a strong decisionmaker would – a provisional restriction that we rationalize further below – but to be able to depart from the plan if it is in his self-interest. Third, we view the type of the policymaker, strong or weak, as unobserved by the private sector. However, the inflation action π reveals useful information about policymaker type.

Each of the periods of our discrete time model is broken into four subperiods, as in displayed in panel A of Table 1. Just prior to the start of the period, a publicly observable policymaker transition may occur: the existing policymaker is continued with probability η and there is a replacement with probability $1 - \eta$.[8] If replacement occurs, then there is strong policymaker with probability γ and a weak one with probability $1 - \gamma$, but the type of the new policymaker is not observed. At the start of the period, an inflation plan p is formulated. In the middle of the period, the private sector forms inflation expectations e. In the end of the period, the inflation action π is chosen. Panel B of Table 1 also shows the two types of central banks. The strong central bank always delivers inflation action at the planned level: $\pi = p$. The weak central bank may choose to depart from the plan, selecting $\pi = d$, or it may choose to mimic, selecting $\pi = p$.

2.1 Expectations and credibility

Our strong central bank is concerned with managing expectations: it therefore must have a model of how expectations will respond to its inflation action p and depend on long-term credibility ρ. We call this the *expectations function*,

$$e(p, \rho). \tag{1}$$

Ultimately, we require that this expectations function is a rational one: it must be consistent with the information, technical, and behavioral structure of the economy.

It is frequently the case that discussions of the management of expectations involve considerations of credibility. In our model, as stressed above, there are actually two notions of credibility that are important; these notions are displayed in Table 2. First,

[8] Ball [1995] argues that the critical construct is the regime, rather than the policymaker. In future research, it seems feasible and desirable to study settings in which observed policymaker changes are noisy signals about regime changes that can occur with or without shifts in policymakers. In the current analysis, though, we assume that policymaker and regime shifts are identical.

there is the likelihood that the decisionmaker at date t will actually choose $\pi = p$: we denote this likelihood as ψ and call it *short-term credibility*. A central bank also must have a model of how its actions affect its short-term credibility: we call this the *short-term credibility function,*

$$\psi(p, \rho), \tag{2}$$

and also impose a rational expectations requirement on this function below. Second, there is the start-of-the-period likelihood that the central bank is of the strong type, which we denote as ρ and call *long-term credibility*. This probability is a key state variable of various models that we study in our research on the management of expectations.

We think that the distinction between short-term and long-term credibility is an important one conceptually and empirically. In the framework that we develop below, short-term credibility is relevant for date t expected inflation and the Phillips curve slope, while ρ would be relevant for longer-term expected inflation and the term structure of interest rates in extensions of the framework.

Bayes' law implies that long-term credibility evolves according to

$$\rho_{t+1} = \begin{cases} \kappa(p_t, \rho_t) = \rho_t / \psi(p_t, \rho_t) & \text{if } \pi_t = p_t \text{ and no replacement} \\ 0 & \text{if } \pi_t \neq p_t \text{ and no replacement} \\ \gamma & \text{if replacement} \end{cases} \tag{3}$$

The notation $\kappa(p, \rho)$ is meant to suggest that long-term credibility is a type of capital for the economy [9] To minimize notational clutter, we sometimes write this equation as $\rho' = \kappa(p, \rho)$ when considering the non-replacement dynamics. The form of (3) emphasizes that policymaker replacement wipes out existing long-term credibility and resets the value to γ.

[9] In perhaps more familiar terms, letting $\tau = s$ be the event in which the decision-maker is of the strong and $\tau = w$ be the event in which the type is weak, then Bayes' rule is that

$$pr(\tau = s | \pi = p) = \frac{pr(\tau = s)}{pr(\tau = s) * pr(\pi = p | \tau = s) + pr(\tau = w) * pr(\pi = p | \tau = w)}$$

with the strong decision-maker always takes the action p, so that $pr(\pi = p | \tau = s) = 1$ **and** $pr\,(\tau = s, \pi = p) = pr\,(\tau = s)$**.**

2.2 Strong versus weak central banks

Strong and weak central banks do not differ in momentary objective, but do in two other ways. First, a strong central bank chooses its inflation action before expectations are formed and a weak central bank chooses its inflation action after expectations are formed. Thus, behavioral differences will arise even in a static model. Second, strong and weak central banks are assumed to differ in terms of how they discount the future. The strong bank has a fixed discount factor of b, which is known to all participants in the economy. The weak central bank has a fluctuating discount factor of β, which is independently and identically distributed according to a continuous probability distribution function $F(\beta)$ with support $(0, b)$. Each period, the weak central bank receives a draw of this discount factor, which is private information. There are two reasons for assuming a random discount factor, one substantive and one technical.

Substantively, we think that a weak monetary authority is one that does not have the internal organizational capital to announce and carry through an inflation plan within a period. When actual decision-makers have such difficulties, they appear to make trade-offs between the present and the future that are short-sighted and vary sharply through time. Our assumption of a random discount factor accords with this observation, in that $E(\beta) < b$ and β is stochastic.

Technically, the continuously distributed random discount factor permits us to generate mimicking, short-term credibility, and expectations functions which depend smoothly on the inflation plan and the state of the economy. More specifically, a strong central bank views expected inflation as a smooth function of its inflation plan, $e(p, \rho)$. When it optimizes, it chooses an inflation plan that is optimal given the state of the economy, $p(\rho)$.

2.3 Central bank policymaking

We now discuss policymaking by the two types of central banks, strong and weak: each makes inflation decisions so as to maximize the present value of the strictly concave momentary objective $w(\pi, e)$, which depends positively on inflation and negatively on expected inflation.

2.3.1 Strong central bank policymaking

The strong monetary authority maximizes welfare taking into account the influence that its inflation plan has on the likelihood and nature of discretionary policy. The Bellman equation for its decision problem is

$$W(\rho) = \max_{p}\{w(p,e) + b\eta W(\rho') + b(1-\eta)\gamma W(\gamma) + b(1-\eta)(1-\gamma)V(\gamma)\}. \qquad (4)$$

The maximization takes place subject to (1),(2), and (3): the rational formation of private sector inflation expectations, $e(p, \rho)$; the short-term credibility associated with mimicking by the weak central bank, $\psi(p, \rho)$; and the evolution of long-term credibility, $\rho' = \kappa(p, \rho) = \rho/\psi$.

In addition to the momentary objective, the right hand side of the Bellman equation includes the discounted values of future welfare if (i) the strong central bank is not replaced, $W(\rho')$; (ii) replacement by a future strong central bank occurs, $W(\gamma)$; and (iii) replacement by a future weak central bank occurs, $V(\gamma)$. That is, our policymaker cares about the state of the economy under other future policymakers, of both his own and other types.

The solution to this maximization includes a time-invariant decision rule $p^*(\rho)$ for inflation that takes into account the management of expectations and the evolution of long-term credibility. Notice that the sole state variable of our model is long-term credibility: this reflects our restricted focus on "minimal state variable equilibria" from a rational expectations modeling perspective, using the terminology of McCallum [1983], and "Markov perfect equilibria" from a game theory perspective, using the terminology of Maskin and Tirole [2001].[10]

2.3.2 Weak central bank policymaking

The weak central banker has two levels of his decision problem. First, he decides on an optimal rate of inflation to set if he behaves in a discretionary manner, d. Second, he decides whether it is desirable to behave in a discretionary manner ($\delta = 1$) or to mimic ($\delta = 0$).

[10] Operationally, we construct equilibria by backward induction from a one period model. When we discuss results of infinite horizon model below, it is thus actually an approximation with a very long, finite horizon. See section 6 for some additional discussion of computation.

The Bellman equation takes the form

$$
\begin{aligned}
V(\rho,\beta) &= \max_{\delta,\pi}\{(1-\delta)M+\delta D\} \qquad (5)\\
M &= [w(p,e)+\beta\eta V(\rho')+\beta(1-\eta)\gamma W(\gamma)+\beta(1-\eta)(1-\gamma)V(\gamma)]\\
D &= [w(\pi,e)+\beta\eta V(0)+\beta(1-\eta)\gamma W(\gamma)+\beta(1-\eta)(1-\gamma)V(\gamma)]\}
\end{aligned}
$$

with $\rho' = \rho/\psi(p,\rho)$ and p is the maximizer of (4). The first bracketed expression, M, captures the welfare if the weak central banks *mimics* its strong counterpart by choosing $\pi = p$. The second bracketed term, D, captures the welfare if it *departs* from the plan and chooses $\pi \neq p$. Notice that by acting in a discretionary manner, the weak central bank has zero credibility in the future unless a regime-replacement event occurs. The right hand side of the Bellman equation contains expressions $V(\rho)$, reflecting our interest in notational simplicity. We define $V(\rho)$ as $E_\beta V(\rho,\beta)$, i.e., as the expectation over the possible realizations of the discount factor.

The decision rules resulting from this optimization problem have several important features. First, the optimal inflation action by the weak central bank simply involves maximizing momentary utility, $w(\pi,e)$: let's call that optimal action d. Second, there is a critical discount factor, $\widehat{\beta}$, such that M and D are equated, given by

$$\widehat{\beta} = \frac{[w(d,e)-w(p,e)]}{\eta[V(\rho/\psi)-V(0)]} \qquad (6)$$

The numerator component is the one-period gain from behaving in a discretionary manner, sometimes called the *temptation,* so that we define $T = w(d,e) - w(p,e)$. The denominator is the expected cost of being identified as a decision-maker of weak type, sometimes called the *punishment*, so that we defined $P = \eta[V(\frac{\rho}{\psi}) - V(0)]$. For values $\beta < \widehat{\beta}$, then, the temptation exceeds the punishment and it is optimal to have an inflation rate higher than planned. For values $\beta > \widehat{\beta}$, it is optimal to carry out the inflation plan. It results in a simple trigger strategy such that $\pi = d$ and $\delta = 1$ if $\beta \leq \widehat{\beta}$ and $\pi = p$ and $\delta = 0$ if $\beta > \widehat{\beta}$. In deciding on when to deviate and how to deviate from an announced inflation plan, a weak central bank treats inflation expectations as predetermined and, more specifically, as a function of the long-term credibility ρ and the inflation plan p that has already been announced, $e(p,\rho)$.

2.4 Rational expectations equilibrium

We now indicate rational expectations restrictions and other consistency conditions that apply for each date t. First, short-term credibility is

$$\psi(p,\rho) = \rho + (1-\rho)m(p,\rho), \tag{7}$$

where m is the probability that a weak central bank will mimic a strong one by choosing $\pi = p$. Second, imposing one layer of rational expectations, expected inflation is a credibility-weighted average of the two possible outcomes for inflation, the strong central bank's plan and the discretionary action by the weak central bank.

$$e(p,\rho) = \psi(p,\rho)\ p + (1-\psi(p,\rho))\ d(e(p,\rho)) \tag{8}$$

Third, imposing another layer of rational expectations, the mimicking function must be consistent with the actual likelihood of deviations, i.e.,

$$m(p,\rho) = 1 - F(\frac{T}{P}), \tag{9}$$

where the temptation T and punishment P were discussed above.

These three conditions are restrictions on three "unknown middle sub-period equilibrium functions" $e(p,\rho), \psi(p,\rho)$ and $m(p,\rho)$. Given these functions, the optimization of the strong policymaker determines $p(\rho)$, which implies equilibrium functions $e(\rho)$, $\psi(\rho)$, and $m(\rho)$ as well as the transition rule, $\rho' = \rho/\psi(\rho)$ in line with (3).

3 Expectations management in a static model

To exposit the most basic ideas about expectations management, we specialize to a simple static framework familiar from the early work of Kydland and Prescott [1977] and Barro and Gordon [1983a] as well as textbook presentations like that of Walsh [2003]. With a single period, there will be no mimicking, so that $\psi = \rho$.

The central bank – strong or weak – maximizes a social welfare function,

$$u(\pi, x, e) = u^* + \omega_1[x - x^*] - \frac{1}{2}\{\pi^2 + \omega_0[\pi - e]^2 + \omega_2[x - x^*]^2\} \tag{10}$$

in which π is inflation, x is the level of output and x^* is the first best level of output.

The central bank prefers higher real activity (x), but dislikes departures of inflation (π) and real activity (x) from first-best values of $\pi = 0$ and $x = x^*$, as well as inflation surprises. The weights $\omega_i \geq 0$ express relative importance of these effects, with the weight of the inflation departures normalized to 1. The parameter u^* captures the first-best level of welfare, but plays no substantive role in the analysis below.

The private sector is simply described by link between surprise inflation $\pi - e$ and real activity in the style of Lucas [1973] and Fischer [1977],

$$x = \alpha(\pi - e), \tag{11}$$

This simple specification is used in many discussions of commitment and discretion in monetary policy, such as Persson and Tabellini [1990].

Following the literature, we will treat inflation as the decision variable for the central bank, so that it is desirable to combine (10) and (11) to produce

$$w(\pi, e) = u^* + \omega_1[\alpha(\pi - e) - x^*] - \frac{1}{2}\{\pi^2 + \omega_0[\pi - e]^2 + \omega_2[\alpha(\pi - e) - x^*]^2\} \tag{12}$$

To maximize this reduced form objective, it is frequently convenient to use the following first order condition,

$$-\pi + (1 - \Delta)[\alpha\omega_1 - (\omega_0 + \omega_2\alpha^2)(\pi - e) + \alpha\omega_2 x^*] = 0 \tag{13}$$

where Δ is the partial derivative of expected inflation e with respect to actual inflation π, $\Delta = \partial e/\partial\pi$, when a particular expectations function has been imposed.

3.1 Pure Discretion

A weak central bank takes its action after the private sector, so that it cannot manage expectations at all ($\Delta = 0$). Using (13), the central bank's decision rule is

$$d = d(e) = \frac{\varphi}{1+\varphi}e + \frac{1}{1+\varphi}B \tag{14}$$

with $\varphi = \omega_0 + \omega_2\alpha^2$ and $B = \alpha\omega_1 + \alpha\omega_2 x^*$. Thus, the discretionary inflation policy partly accommodates expectations of inflation and partly tries to stimulate the economy because output is inefficiently low, as stressed in the literature.

Imposing rational expectations ($e = d$), the equilibrium inflation rate under pure discretion is

$$d = \omega_1 \alpha + \omega_2 \alpha x^* = B, \tag{15}$$

so that the compound parameter B is the celebrated "inflation bias under discretion".

3.2 Uncertainty about commitment or discretion

We are interested in environments in which private agents do not know the type of the central bank – whether it is strong or weak – when they form expectations. In line with our discussion in section 2, we analyze this basic static model by working backwards from the actions of a weak central bank, should one exist, choosing inflation after expectations have been formed.[11]

3.2.1 End: The weak central bank

The weak central bank takes its action after expectations are formed, so that $\Delta = 0$ as in the analysis of pure discretion above. Using the first order condition, we find that the weak central bank's decision rule is unchanged from (14), $d = \frac{\varphi}{1+\varphi} e + \frac{1}{1+\varphi} B$.

3.2.2 Middle: Expectations and discretion

Private agents form expected inflation e knowing that there is some chance that there will be a strong central bank in place (ψ), making inflation $\pi = p$, and some chance that there will be a weak central bank in place ($1 - \psi$), making inflation $\pi = d$. Hence, expected inflation is $e = \psi p + (1 - \psi) d$ in line with (8). The equilibrium inflation action of the weak central bank arises as a fixed point between expectations and actions. In the current context, the degree of short-term credibility influences the nature of this fixed point, since it affects the relative weight placed on the strong central bank's plan and the weak central bank's action. The equilibrium weak central bank action is

$$d(p;\psi) = \frac{\varphi\psi}{1+\varphi\psi} p + \frac{1}{1+\varphi\psi} B = \theta p + (1-\theta) B \tag{16}$$

with B defined in (15) as the inflation bias if there is a weak central bank with certainty (that is, if $\psi = 0$).

[11] Since there is no mimicking, long-term and short-term credibility are identical: we use ψ rather than ρ in this section because that is the better interpretation in our dynamic analysis.

There is thus an "middle subperiod equilibrium function" $d(p;\psi)$ describing the weak central bank's response to the inflation plan: the intensity of this reaction, which we call θ, is between 0 and 1 so long as $\varphi > 0$, since $\theta = \frac{\varphi\psi}{1+\varphi\psi}$. Further, θ increases with the degree of short-term credibility ψ. Correspondingly, with $\psi > 0$ and thus $\theta > 0$, monetary discretion gives rise to a smaller degree of inflation bias – at a given p – because expected inflation is held down by the chance that there is a strong central bank in place.

With one configuration of model parameters, it is not necessary to solve the rational expectations fixed point. If $\varphi = \omega_0 + \omega_2\alpha^2 = 0$ because the objective function weights on quadratic terms involving expectations are zero, then the weak monetary authority simply always chooses $d = B$. In that case, assumed by Barro [1986], Backus and Driffill [1985a,b], and Cukierman and Leviatan [1991], the inflation plan p has no effect on the behavior of the weak central bank.

3.2.3 Start: The strong central bank

We next consider the inflation plan formulated by the strong central bank, which optimizes taking as given the equilibrium expectations rule

$$e(p;\psi) = \psi p + (1-\psi)d = \Delta p + (1-\Delta)B$$

where $\Delta(\psi) = [\psi + (1-\psi)\theta(\psi)]$ and thus $1-\Delta = (1-\psi)(1-\theta)$.

Imperfect short-term credibility ($\psi < 1$) implies that the strong central bank cannot perfectly manage expectations ($\psi < 1$ implies $\Delta < 1$): a part of expectations is invariant to its plan. However, in its expectations management, the strong central bank takes into account the direct effect of its inflation plan on expectations (ψp) and also the indirect effects that operate through the plan's influence on private sector beliefs about the weak central bank's inflation action, as captured by $(1-\psi)\theta p$.

Using (13), the optimal inflation plan takes the form

$$p(\psi) = [1 - \frac{\Delta(\psi)}{1+\varphi(1-\Delta(\psi))^2}]\ B. \tag{17}$$

There are a number of interesting aspects of the strong central bank's inflation plan in a static setting of imperfect short-term credibility. First, there is some inflation bias unless credibility is complete ($\psi = 1$): a strong central bank does not fully offset the inflation bias from expectations about the weak central bank's behavior, because it would face losses from doing so due to unexpected inflation. Second, the coefficient attached to the bias term B coefficient has the property that it is 1 when $\psi = 0$ (no short-term credibility) and that it is 0 when $\psi = 1$ (full short-term credibility). That is, a strong central bank with low short-term credibility will behave very much like a weak central bank in terms of its inflation plan.

Given the inflation plan just determined, the weak central bank's behavior is

$$d(\psi) = \theta(\psi)p(\psi) + (1 - \theta(\psi))B = p(\psi) + \frac{\Delta(\psi)(1 - \theta(\psi))}{1 + \varphi(1 - \Delta(\psi))^2}B \tag{18}$$

Thus, in equilibrium, the weak authority chooses a higher rate of inflation than the inflation plan, although the effect is minor when short-term credibility is low (because the coefficient on B is close to zero in that setting).[12]

3.3 Inflation and real activity

Given the expectations function, $e = \psi p + (1 - \psi)d$, it is now easy to determine the behavior of real activity as $x = \alpha[\pi - e] = \alpha[\pi - \psi p - (1 - \psi)d]$. This simple expression implies that there is a recession when there is a strong central bank in place,

$$x^s = -\alpha(1 - \psi)(d - p)$$

since $d > p$. This recession will be small when short-term credibility is very high (since $\psi = 1$), as well as when it is low (since $d - p$ is small because the coefficient on B in (18) is close to zero). By contrast, when there is a weak central bank in place, there is a boom

$$x^w = \alpha\psi(d - p).$$

This is a small effect when short-term credibility is small (ψ close to 0) but it is increasing as short-term credibility increases, both because the "slope" $\alpha\psi$ increases and because the size of $(d - p)$ increases. Hence, higher short-term credibility benefits the weak central bank when he produces an inflation surprise.

[12]It also chooses a lower rate of inflation than under full information, as seen in (16) unless the nonnegative response coefficient θ is zero. This occurs only when expected inflation enters linearly in the reduced form objective, as assumed by Barro [1986], Backus and Driffill [1985a,b] and Cukierman and Leviatan [1991], and requiring $\omega_0 = \omega_2 = 0$ in our model.

3.4 Effects of imperfect credibility

Figure 1 provides an illustration of the general effects imperfect short-term credibility using "Case 1" parameters presented in Table 3 and discussed further below: these parameters make inflation bias B equal to .2 (20% per year). Panel A shows how the response coefficient governing discretionary inflation, $d = \theta p + (1-\theta)B$, and that governing expected inflation, $e = \Delta p + (1-\Delta)B$, depend on short-term credibility ψ. The strong central bank has more leverage on expectations than on discretionary inflation, of course, because $e = \psi p + (1-\psi)d$: it controls a greater part of expected inflation directly via its plan when its credibility is higher.

The strong central bank's optimal inflation plan at various levels of credibility is shown in panel B, along with expected inflation and discretionary inflation. Notice that a weak central bank chooses a discretionary inflation rate much less than 20% if its credibility is high. The real consequences of the optimal inflation plan are displayed in panel C. In line with the discussion above, the strong central bank produces low output and the weak central bank produces high output, but the degree of these variations depends on credibility.

All in all, this simple model delivers a number of interesting implications about an environment of imperfect credibility, as follows:

Management of expectations: A strong central bank in a setting of imperfect credibility will manage the beliefs of agents in part by changing their understanding of what a weak central bank will do;

Imperfect credibility and inflation bias: Even a strong central bank will not eliminate the inflation bias arising from discretionary policymaking if it is in a setting of imperfect credibility;

Failure of inflation plans: Inflation plans will turn out to fail, when central banks turn out to be discretionary in their actions, even though these plans were consistent with optimal management of expectations by a strong central bank;

Real effects of inflation policies: Under imperfect credibility, there will be a real boom when a weak central bank generates a surprised inflation and a real recession when a strong central bank maintains its inflation plan.

Cukierman [1986] and Cukierman and Leviatan [1991] stress the latter three implications. The first implication does not appear in their analysis because the weak central bank's inflation action, which we call d, does not depend on expected inflation because of their assumption about the momentary objective.

4 Managing mimicking

We now consider an extension of the static model to allow for management of endogenously determined short-term credibility. That is, there is now an expectations function of the form

$$e(p,\rho) = \psi(p,\rho)p + [1-\psi(p,\rho)]d(p,\rho) \tag{19}$$

Expected inflation continues to be a weighted average of the inflation rates of strong (p) and weak (d) central banks, but the weight now depends on both the level of long-term credibility and on the inflation plan. Generally, this setup means that part of the effect of an inflation plan is on mimicking incentives. Specifically, a lower inflation plan can consequently have a heterodox effect on expected inflation, raising it because the likelihood of discretion increases.

To generate endogenous mimicking within a static setting in ways that are instructive about our full dynamic model with random discounting, we suppose that a weak monetary authority faces a utility cost of ξ if it deviates, with this fixed cost being drawn from a continuous distribution F with support $[0,\Xi]$. Accordingly, an authority will deviate if the momentary gain from deviating (temptation) exceeds the cost, i.e., $T(p,e) = w(d,e) - w(p,e) > \xi$ and otherwise it will mimic, so that the mimicking function is

$$m = 1 - F(T(p,e))$$

which is decreasing in temptation.

4.1 An analytical example

We can generate a convenient example by assuming that the reduced form objective function w is quasi-linear in expected inflation ($\omega_0 = \omega_2 = 0$), so that weak central bank's decision rule is just $d = \omega_1\alpha = B$. Given the discretionary inflation rate, it then follows that the welfare under discretion is $w(B,e) = u^* - \frac{1}{2}B^2 + \omega_1[\alpha(B-e) - x^*]$ and that under the inflation plan is $w(p,e) = u^* - \frac{1}{2}p^2 + \omega_1[\alpha(p-e) - x^*]$, so that the gain from deviating is

$$T(p,e) = w(B,e) - w(p,e) = -\frac{1}{2}(B^2 - p^2) + \omega_1\alpha(B-p) = \frac{1}{2}(B-p)^2. \tag{20}$$

Hence, the mimicking depends only on the inflation plan, $m = 1 - F[\frac{1}{2}(B-p)^2]$, with some interesting properties. First, if planned inflation is zero, then mimicking will be positive so long as there are sufficiently high mimicking costs, $\Xi > \frac{1}{2}B^2$. Second, with p increasing toward B, the gain from deviating decreases, so that the weak central bank is more likely to carry out the planned inflation. In fact, mimicking becomes certain when $p = B$. Third, if the distribution of costs takes the form $F(T) = (T/\Xi)^{1/2}$, then mimicking depends linearly on the gap between discretionary and planned inflation,

$$m = 1 - \frac{1}{\sqrt{2\Xi}}(B-p), \tag{21}$$

which is increasing for p between 0 and B as plotted in panel A of Figure 2.[13]

In turn, the plan also affects the short-term credibility, since $\psi(p,\rho) = \rho + (1-\rho)\, m\,(p)$:. a higher planned inflation rate is more credible because it is more likely to be mimicked. This effect is weighted by $(1-\rho)$, so the impact of p on ψ is particularly strong when long-term credibility ρ is low (the dashed line in panel B of Figure 2).

Having this optimal response of ψ to p in hand, we now turn to its consequence for the expected inflation. Since inflation expectations take the form (19) and the weak central bank is just an automaton setting $d = B$, the effect of a marginally higher inflation plan is

$$\frac{\partial e(p,\rho)}{\partial p} = \psi - (B-p)\,\frac{\partial \psi(p,\rho)}{\partial p} = \psi - (1-\rho)\,(B-p)\,\frac{\partial m(p)}{\partial p} \tag{22}$$

The first term is simply the direct effect of the plan on expectations: a marginally higher inflation plan raises expected inflation. However, there is an offsetting effect captured by the second term, which is new to the current setup: it reflects the endogenous response of the likelihood that the plan will be carried out, i.e., the short-term credibility ψ. We have seen above that a higher rate of the inflation plan raises the mimicking rate so that $\frac{\partial m(p)}{\partial p} > 0$. Since the discretionary inflation rate B is always higher than the inflation plan p, it follows that this second channel involves a heterodox effect: a higher planned inflation rate lowers expected inflation because it raises the likelihood that the plan will be carried out. This second term is weighted by

[13] Figure 2 shows the key variables as a function of the plan p, for three levels of credibility: $\rho = .2$ (solid line), $\rho = .5$ (dashed line), and $\rho = .8$ (dashed and dotted line). All the functions are computed using the Case 2 parameter values in Table 3.

the probability that there is a weak type of central bank in place $(1-\rho)$ and by the gap between a discretionary and planned inflation $(B-p)$. Thus, this unconventional effect of the inflation plan is strengthened with a more aggressive planned inflation rate, as well by a lower level of central bank's long-term credibility ρ.

Under our specific assumptions on the objective and the distribution, we are able to obtain a closed form expression that enables us to easily determine which of these opposite forces dominates:

$$\frac{\partial e(p,\rho)}{\partial p} = 1-(1-\rho)\,(B-p)\sqrt{\frac{2}{\Xi}}$$

When long term credibility ρ is high, the mimicking effect is less important, because agents assign a low probability $(1-\rho)$ that the central bank in place is weak. Thus, the direct effect of the plan p dominates (as shown by both the dashed line and the dashed-dotted line in panel C of Figure 2). However, for low long-term credibility $\rho < 1-\sqrt{\frac{\Xi}{2B^2}}$, the function $e\,(p,\rho)$ exhibits a U-shape[14], with the slope being negative for low p ($p < \sqrt{\frac{\Xi}{2(1-\rho)^2}} - B$) and positive for p closer to B (as shown by the solid line in panel C of Figure 2). This nonmonotonicity makes managing mimicking a tricky task. In particular, for an aggressively low planned inflation rate p, a further slashing has the unconventional effect of increasing expected inflation e.

The quasi-linear objective function also makes it possible to write unexpected inflation as a quadratic function of the inflation plan:

$$\begin{aligned} p-e(p,\rho) &= (1-\psi(p))(p-B) = (1-\rho)(1-m(p))(p-B) \\ &= -(1-\rho)\frac{1}{\sqrt{2\Xi}}(p-B)^2. \end{aligned}$$

which is always negative. The Phillips curve translates unexpected inflation into output: $x = \alpha(p-e(p,\rho))$. So, carrying out the inflation plan always leads to recessions, as shown by panel D of Figure 2. The recession is deeper with a lower long-term credibility ρ since the *ex ante* likelihood of the central bank being a weak type $(1-\rho)$ dictates the scale of unexpected inflation.

Optimal inflation for the strong central bank is found by maximizing $w(p,e) =$

[14]A technical assumption to obtain the U-shape with positive ρ is: $\Xi < 2B^2$. This condition together with the one ensuring positive mimicking $\Xi > B^2/2$ define the admissible range for parameter Ξ.

$u^* - \frac{1}{2}p^2 + \omega_1 [\alpha(p-e) - x^*]$, with the unexpected inflation function given above. The optimal plan is

$$p^* = \frac{\alpha\omega_1(1-\rho)}{\sqrt{\Xi/2} + \alpha\omega_1(1-\rho)} B$$

which is represented in the panels of Figure 2 by "*", for the equilibrium level of mimicking probability $m(p^*)$, short term credibility $\psi(p^*, \rho)$, expected inflation $e(p^*, \rho)$, and recession generated by following the plan $x^c(p^*, \rho)$.

Hence, with imperfect information about type ($\rho < 1$), a strong central bank will choose an inflation plan that does not fully eliminate the inflation bias, even when it can manage expectations, echoing the results of section 3. Furthermore, relative to a monetary authority who faces exogenous short-term credibility, the strong decision-maker will choose a higher planned rate of inflation[15]. He knows that tightening the inflation plan has a diminishing effect on expected inflation, because the probability of hitting such a tough inflation rate declines. With a low long-term credibility, a too aggressive plan could even move the expected inflation against the central bank, rising the output losses dramatically when he sticks to his plan. Hence, the endogenous reaction of ψ to p sets a barrier for the strong central bank in selecting a low inflation plan, especially when the long-term credibility is at a low level.

4.2 A more general setting

The previous section makes specific assumptions on the objective $w(\pi, e)$ and the distribution $F(T)$, in order to obtain a closed form solution for the expected inflation's reaction to both the long term credibility ρ and the inflation plan p. When we incorporate a quadratic term on output in $w(\pi, e)$ as in section 3, we preserve and even strengthen the effects just discussed.

The quadratic term on output brings about two key differences from the analytical example: (i) the discretionary inflation rate is no longer constant at B, but reacts to the expected inflation $d(e)$ as in (14); (ii) the expectations can no longer be

[15] The first order condition for the optimal plan is:

$$p^* = B(1-\psi) + B(B-p)\frac{\partial\psi}{\partial p}$$

with ψ reacting positively with the inflation plan. Therefore, the endogeneity of ψ adds the positive second term to the determination of optimal plan, which would be just $B(1-\psi)$ if short-term credibiltiy ψ is exogenous.

eliminated from temptation so that $T(p,e) = w(d,e) - w(p,e)$, in contrast with (20). These two elements introduces a fixed point problem in computing equilibrium expected inflation $e(p,\rho)$. Given any plan p and expected inflation e, the mimicking function is $m(p,e) = 1 - F(T(p,e))$, which leads directly to short-term credibility by $\psi(p,e,\rho) = \rho + (1-\rho)\,m(p,e)$. So the equilibrium expected inflation at any level of inflation plan p and long-term credibility ρ is the solution to:

$$e(p,\rho) = \psi(p,e,\rho)\,p + (1-\psi(p,e,\rho))\,d(e)\,.$$

The fixed point problem prevents us from having closed form solution for the more general setting. Nevertheless, we can obtain the marginal effect of the inflation plan on the equilibrium inflation expectations, taking into account that the expected inflation e on the right hand side of the equation is also a function of p:

$$\frac{\partial e(p,\rho)}{\partial p} = A(p,\rho)\left[\psi - (d-p)(1-\rho)\frac{\partial m(p,e)}{\partial p}\right]$$

The two terms in bracket are the marginal effect of the plan on inflation expectations in the analytical example (22), only with B replaced by d. Therefore, the two offsetting impacts of the inflation plan discussed above remain the same, further amplified by the multiplier $A(p,\rho) = \{1 + (1-\rho)\,[(d-p)\,\frac{\partial m(p,e)}{\partial e} - (1-m)\,\frac{\partial d(e)}{\partial e}]\}^{-1} > 1$.[16] Thus, the heterodox effect of the inflation plan on expected inflation, introduced by endogenous mimicking, is even stronger in the more general model and particularly so when long-term credibility is low.[17]

[16] To see why this expression is greater than one, first notice that $\frac{\partial m(p,e)}{\partial e} < 0$ because the incentive to carry out a plan declines when expected inflation rises, holding everything else constant. The discretionary inflation rate is always higher than the planned rate so that $(d-p)\,\frac{\partial m(p,e)}{\partial e} < 0$. Additionally, if the expected inflation increases, the optimal reaction of discretionary inflation will also rise according to (14). Therefore, $\frac{\partial d(e)}{\partial e} > 0$ and the whole expression in the **squared** bracket is negative.

[17] That is, since $A(p,\rho) = \{1 + (1-\rho)\,[(d-p)\,\frac{\partial m(p,e)}{\partial e} - (1-m)\,\frac{\partial d(e)}{\partial e}]\}^{-1}$, we see that $A(p,\rho)$ is greater than one if $\rho < 1$ because the bracketed term is negative. Further, for the same reason, A becomes larger as $(1-\rho)$ becomes larger, i.e., at lower levels of long-term credibility.

5 Reports, rules and inflation commitments

We have so far depicted a strong monetary authority who knows that private agents will respond to his announced plan, but also recognizes that it is only partially credible. We now reanalyze the announcement, with specific emphasis on the market interpretation of central bank signals and the role of transparency in central bank management of expectations.

5.1 Signalling equilibria and inflation reports

Our approach to this point has been to restrict attention to a rational expectations equilibrium of a Markovian form, rather than to consider an explicit game between the central bank and the private sector. We now analyze a signalling game, with the central bank being the sender of the message p and a large number of private agents being the receivers of the message. Since the central bank is of unknown type, the relevant equilibrium concepts is be the sequential equilibrium of Kreps and Wilson [1982] or the perfect Bayesian equilibrium of Fudenberg and Tirole [1991]. While there are subtle differences in these approaches, we will treat them as equivalent for our purposes.

Under the direct game-theoretic approach, the central problem is that any signal — any value of p — can be an equilibrium so long as it is supported by sufficiently pessimistic out-of-equilibrium beliefs. It is clear that the only equilibrium of the game in our economy is a pooled one, in which both types of central banks send the same message: if they send different ones, so that they are distinguished, then the weak central bank can always improve its welfare by sending the same message as the strong bank. However, rather than the value p^* which we determined above, suppose that an alternative pooled equilibrium $p = a$ were considered. Then, if private agents interpret other messages as indicating that central bank type is weak, then any $p = a$ is indeed an equilibrium. This bewildering set of equilibria has led many to avoid modeling settings with private information and partial commitment, features which appear to us to be essential to the analysis of the management of expectations and the evolution of credibility.

The announcement game in our model is unusual in two dimensions: (i) the strong authority generates a publicly observable inflation plan as a by-product of his action; and (ii) imitation is optimal for the weak authority. It turns out that many of the refinements that have been developed to reduce the set of equilibria in signalling games do not "work" in this context of costless imitative signalling. Notably, the intuitive criterion of Cho and Kreps [1987] produces the counter-intuitive conclusion that the only signalling equilibrium involves full inflation bias, $\pi = p = B$.[18] There is no equilibrium at all under the "perfect sequential equilibrium" approach of Grossman and Perry [1986], who introduce the idea of using Bayesian reasoning to discipline out-of-equilibrium beliefs, but in a manner that imposes only weak coherency on these beliefs.

However, the approach of Mailath, Okuna-Fujiwara, and Postelwhaite [1993], which imposes strongly coherent out-of-equilibrium beliefs of a Bayesian form, can be used to show that the unique equilibrium is exactly the p^* that we have derived above. There are three essential features of that approach. First, any out-of-equilibrium message must be one that is sent in an alternative sequential equilibrium by some set of agents. Second, the incentives that various types of agents have to send an alternative message m are evaluated by comparison of benefits in a candidate and alternative equilibrium. This comparison induces coherency restrictions on beliefs, which specify an out-of-equilibrium probability distribution of sender type. Third, if these coherent beliefs are inconsistent with the support of the candidate equilibrium, it is "defeated" by the alternative and, hence, discarded.

To study equilibrium selection along these lines, we use a special case of the model of section 3, with quasilinear utility function. This special case allows us to produce simple graphical description of the key ideas, for three reasons. First, the reduced form objective may be written as $w(\pi, e) = -\frac{1}{2}\pi^2 + B(\pi - e)$, where $\pi = p$ or $\pi = d$. An indifference curve in (p, e) space is thus $e = p - \frac{1}{B}w - \frac{1}{2B}p^2$ for the strong central bank and $e = d - \frac{1}{B}w - \frac{1}{2B}d^2$ for the weak central bank. Second, the quasilinear objective implies that $d = B$, as discussed above, so that the weak central bank has indifference curves that are horizontal in (p, e) space. Third, for any belief $\widetilde{\psi}$, there is a corresponding point $\widetilde{e} = \widetilde{\psi}p + (1 - \widetilde{\psi})B$ in (p, e) space. As a related matter, we can

[18] Cukierman and Leviatan [1991, appendix] also study a signalling game in a setting with strong and weak policymakers. They use reasoning along Cho-Kreps line but depart from the standard inutive criterion approach by assuming that message-sending incentives are evaluated in an alternative pooled equilbrium, which anticipates aspects of the approach of Mailath et al [1993]. However, the method of Mailath et al. is more generally applicable to analysis of economies with more than two types, as with our random discount factor or random cost models. Moreover, as we stress in King, Lu, and Pasten [2007], a direct application of the Cho-Kreps approach leads to selection of the worst equilibrium, rather than the optimal one.

write the locus of pooled equilibrium points as $e = \psi p + (1-\psi)B$, corresponding to the objective probability ψ. We structure our discussion around of Figure 5; in each panel, the horizontal axis is the inflation plan (p) and the vertical axis is expected inflation (e).

5.1.1 Graphical determination of p^*

In panel A, indifference curves for the strong central bank are drawn in with dashed lines: in this panel and below, since $e = p - \frac{1}{B}w - \frac{1}{2B}p^2$, the indifference curve is increasing and strictly concave on the interval interval (0,B) as shown in the figure. Higher levels of the central bank objective correspond to lower expected inflation at any fixed p.

The solid line in each panel is the locus of pooled equilibrium points: expected inflation is $e = \psi p + (1-\psi)d$ with $d = B$ resulting from quasi-linear utility. Panel A thus shows the determination of p^* graphically, corresponding to the algebraic derivation in section 3.

5.1.2 Multiplicity of equilibria

Any point a on the solid line can be a Bayesian (pooled) equilibrium of the signalling game, if agents believe that $p \neq a$ implies that the central bank type is weak. We label out-of-equilibrium beliefs as $\widetilde{\psi}(p)$, here and below, so that the specific assumption is that $\widetilde{\psi} = \psi$ at $p = a$ and $\widetilde{\psi} = 0$ otherwise, so that $e = B$ if $p \neq a$ in panel B of the figure. Neither type of central bank is better off in the inflation bias equilibrium with $p = B$ than with more moderate inflation at $p = a$. Further, since an off-equilibrium signal will never be sent, it is the case that these off-equilibrium beliefs cannot be invalidated. That is: it is possible for signalling to lead to an "expectation trap" in this one period model, when there is latitude to specify out-of-equilibrium beliefs.[19]

5.1.3 Disciplining off-equilibrium beliefs

Refinements of signalling equilibria all necessarily involve the interpretation of out-of-equilibrium messages and the incentives that various parties would have to send such messages, even though these messages would never be sent in equilibrium. If

[19] Chari, Christiano, and Eichenbaum (1998) define an "expectation trap" of a different sort, in a discretionary equilibrium rather than a signalling game.

an out-of-equilibrium message is contemplated, then its implication for belief about types should not be inconsistent with the incentives that types have to send such a message. However, if no type of agent would ever find it desirable to send a message, then no restrictions are placed on beliefs, so that these can be structured to support an equilibrium.

While most refinement criteria embody this general logic, individual refinements differ on how the benefit of sending an out-of-equilibrium message is evaluated. Cho and Kreps [1987] develop an intuitive criterion which specifies that one should use the sender's favorite beliefs to compute his reward to sending an out-of-equilibrium message. Grossman and Perry [1986] alternatively argue that the proposed beliefs concerning an out-of-equilibrium message have to agree with the sender's incentives evaluated at those beliefs, a self-referential property that we call weak consistency of beliefs. We find more compelling the approach of Mailath et. al. [1993], which imposes strong coherency on out-of-equilibrium beliefs. Strong coherency requires that the incentives to send an out-of-equilibrium message are evaluated using the beliefs that support such a message as another sequential equilibrium.

Ruling out high inflation equilibria The equilibrium selection approach of Mailath et al. [1993] eliminates candidate equilibria $h > p^*$, as follows. As shown in panel C of Figure 5, h must be supported by beliefs that make the candidate equilibrium less desirable to both strong and weak central banks, i.e., beliefs that lead to expectations which lie above the indifference curves through h for both types. We provide one illustration of beliefs that would support the equilibrium h in the figure, but now show why any such belief is problematic.

In particular, for $p < h$, it must be the case that the weak central bank does not prefer $p < h$, so that agents must assign a probability

$$\widetilde{\psi}(p) < \frac{(B-h)}{(B-p)}\psi < \psi$$

to there being an inflation action of p. This conclusion derives from the fact that a weak central bank prefers lower expected inflation, while always setting its inflation action to $\pi = B$. It thus will not strictly prefer p only if $e_h = \psi h + (1-\psi)B < e_p = \widetilde{\psi}p + (1-\widetilde{\psi})B$, which leads directly to the above inequality.

Now, we ask whether the belief $\widetilde{\psi} < \psi$ which supports h is coherent with message-sender incentives in an alternative equilibrium p^*. We find that both strong and weak central banks would send p^* because each has higher welfare in that alternative equilibrium relative to the h equilibrium.[20] Thus, the Bayesian belief is ψ rather than $\widetilde{\psi} = \frac{B-h}{B-p}\psi < \psi$. Hence, we say that the supporting belief for h is not *strongly coherent* at p^* and we rule out the equilibrium h. In the terminology of Mailath et. al. [1993], the candidate equilibrium h is *defeated* by the alternative equilibrium p^*.

Ruling out low inflation equilibria The equilibrium selection approach also rules out a low inflation plan outcome ($p = l < p^*$) shown in panel D. An equilibrium $p = l$ must be supported by beliefs that make it undesirable for the strong central bank to pick p^*, as illustrated by the dotted line. However, since this expectation lies above the indifference curve (solid line) at p^*, it again involves the belief that a message of p^* is more likely to be sent by a weak bank. That is, letting the point exactly on the strong central bank indifference curve be $\widehat{e}$, we see that $\widetilde{e} = \widetilde{\psi}p^* + (1-\widetilde{\psi})B > \widehat{e} > e^* = \psi p^* + (1-\psi)B$ so that

$$\widetilde{\psi} < \psi.$$

Considering sender incentives once again, we find that the strong central bank would send this message (its welfare is maximized at p^*), while the weak central bank is worse off with higher expected inflation so that it would not. Thus, the candidate equilibrium l is ruled out because the coherent belief is that the message is being sent by a strong central bank, but the restriction on belief necessary to support l is that the message is unlikely to be sent by a strong central bank ($\widetilde{\psi} < \psi$).

Existence A potential difficulty with using "belief-based refinements" along the lines of Grossman and Perry [1986] and Mailath et. al. [1993] is that there can be no equilibrium. We have described so far why there cannot be an equilibrium $h > p^*$ or $l < p^*$ but this does not guarantee that p^* itself is an equilibrium.

However, it is easy to show that there is no existence problem in our setting.

[20] The weak central bank benefits because $e^* < e_h$ and the strong central bank benefits because the indifference curve through (p^*, e^*) lies below that through (h, e_h).

Suppose that there is a out-of-equilibrium belief function of the form

$$\begin{aligned} \widetilde{\psi}(p) &= 0 \text{ for } p < p^* \\ \widetilde{\psi} &= \psi \text{ for } p \geq p^* \end{aligned}$$

Then, we can show there is no conflict with the requirements of Mailath, Okuno-Fujiwara and Postlewaite [1993], as follows.[21]

If the candidate equilibrium is p^* and the alternative equilibrium is $p < p^*$, a weak central would find it desirable to send the message because lower p involves lower expected inflation. By contrast, a strong central bank will not send the message, since its welfare is maximized at p^* given belief ψ and its welfare is even lower when the belief is more pessimistic. Hence, the message $p < p^*$ should be interpreted as coming from a weak central bank, as specified above.

If the candidate equilibrium is p^* and the alternative equilibrium is $p > p^*$, a weak central bank will not send this message since expected inflation is higher ($\psi p + (1 - \psi)B > \psi p^* + (1 - \psi)B$ is implied by $p > p^*$). Similarly, a strong central bank would not send the message, since its welfare is maximized at p^* given objective beliefs ψ. Since neither central bank will send such a message, no restrictions are placed on out-of-equilibrium beliefs.

The bottom line Overall, we have found that p^* is the sole surviving signalling equilibrium.[22] In our analysis of section 3 above, we found that p^* would be the optimal choice of a strong central bank employing a rational expectations modeling approach and we assumed that a weak central bank would also make this announcement. We now have a stronger finding in the context of a signalling game: the strong central bank's optimal announcement p^* is the unique equilibrium that survives the refinement process. In addition, the optimal signalling strategy for the weak central bank is to mimic the same announcement p^*.

[21] There is a conflict with the requirements of Grossman and Perry [1986] which impose a weaker coherency on beliefs. A strong central bank can always find a higher inflation message (say $h > p^*$) that will distinguish him from a weak central bank and that will be desirable if he evaluates the benefits under the assumption that he is distinguished. As a consequence, there is no signalling equilibrium under that belief-based refinement.

[22] We discuss the application of alternative refinements to a simple microeconomic model with many of the same properties of our current framework in King, Lu and Pasten [2007].

5.2 Inflation rules

The game-theoretic perspective also indicates that the announcement plays a central role, perhaps surprisingly because it is not informative about central bank type. In the signalling game, a strong central bank has no influence on private sector expectations unless it sends a message and it has no incentive to choose low inflation unless it has an influence on inflation expectations.[23] Thus, if it can disclose its planned inflation action, then a strong central bank should do so. Lack of transparency, in this sense, would lead to the fully discretionary outcome $\pi = B$ if the strong central bank did not make the announcement. Intrinsic strength is not a substitute for communication within this model.

The view that announcements are central contrasts with an alternative viewpoint which has some credence among monetary policy analysts working from rational expectations perspective, which we summarize as follows. First, the central bank is in an environment with imperfect credibility, but the participants in the model all understand exactly that a disciplined central bank would issue the inflation report $p^*$and that a weak central bank would issue the same report, so that the report is not informative. Second, as a consequence, a disciplined central bank which uses the inflation rule p^*, implemented with an appropriate interest rate or monetary aggregate instrument policy, would also secure the same performance in the absence of the announcement. In the jargon of current policy discussions, the alternative perspective would rationalize a system of *implicit inflation targets* and downplay the importance of announcements of *explicit inflation targets.* However, the basic game-theoretic analysis of this section both supports our focus on p^* as the equilibrium inflation plan and indicates that transparency about inflation plans is critical to their success.

[23]When the central bank moves first but does not inform the private sector about its choice, the game between these two parties is essentially the one with perfect information in which they choose actions simultaneously. (Osborne and Rubinstein 1994, page 202). As is well known, a simultaneous game between central bank and the private sector produces the famous "inflation bias": the expectation of private sector is taken as given when the central bank makes the inflation choice and the private sector has rational expectations.

6 Evolution of long-term credibility

To this point, we have provided basic examples of expectations management. In section 3, we studied how a strong monetary authority with a short-horizon would optimally choose inflation when there was exogenous imperfect credibility, with its inflation plan influencing expected inflation both directly and via its effect on weak central bank's discretionary inflation action. In section 4, we show this strong authority would manage credibility with endogenous mimicking by a weak central bank with random costs of behaving in a discretionary manner. We think that these examples – each of which contains a closed form solution under certain parameters – shed light on important aspects of managing expectations.

However, the simple models do not describe the dynamic evolution of credibility that seems essential to understanding many issues in macroeconomics or the related choice of an inflation plan, set by a strong central bank that is managing its stock of long-term credibility. It is to this topic that we now turn.

6.1 Mechanical dynamics and general principles

Specification of a mimicking function $m(\rho)$ allows us to write *short-term credibility* – the probability that the planned inflation will be carried out this period – as

$$\psi(\rho_t) = \rho_t + (1 - \rho_t)m(\rho_t).$$

This expression makes clear that short-term credibility will be higher than long-term credibility because of mimicking behavior. As stressed in the discussion in section 3 above, it is short-term credibility that plays a role in the determination of output and inflation in the economy at a given point in time.

Further, as discussed in section 2 above, since the observed inflation rate is the only information about the nature of central bank type, then Bayesian learning implies that

$$\rho_{t+1} = \left\{ \begin{array}{ll} \rho_t/\psi_t = \rho_t/[\rho_t + (1 - \rho_t)m(\rho_t)] & \text{if } \pi_t = p_t \\ 0 & \text{if } \pi_t \neq p_t \end{array} \right\}$$

if there is no replacement. That is, Bayes' rule governs the linkage between inflation plans – which influence the rate of mimicking – and the evolution of the state variable of our model, which is long-term credibility.

In terms of general principles, there is thus an interesting interplay between short-term credibility ψ and long-term credibility ρ. If short-term credibility is high (close to one) then long-term credibility evolves very slowly (if $\psi = 1$, then Bayesian learning specifies that $\rho_{t+1} = \rho_t$). This is because there is little information in the observation that inflation is at the planned rate. On the other hand, if short-term credibility is low, then there is a good deal of information in the fact that inflation is at the planned rate. Further, short-term credibility can be high simply because the likelihood of mimicking (m) is high ($\psi = 1$ if $m = 1$ irrespective of the level of ρ). So, if there is substantial mimicking, then long-term credibility will evolve very slowly.

6.2 Computation of equilibria

We employ a straightforward backward induction algorithm to deduce the optimal inflation policy, in essence by iterating on (4) and (5).[24] We start with an one period model, like that of section 3, determining optimal inflation actions by strong and weak central banks ($p_1^*(\rho)$; $d_1(\rho)$); equilibrium outcomes (short-term credibility, $\psi_1(\rho) = \rho$, since there is no mimicking in the last period; expected inflation $e_1(\rho)$); and welfare measures $W_1(\rho)$ and $V_1(\rho)$. We then iterate from this initial condition, constructing ($p_j^*(\rho)$; $d_j(\rho)$); $\psi_j(\rho)$; $e_j(\rho)$); and welfare measures $W_j(\rho)$ and $V_j(\rho)$ from knowledge of $W_{j-1}(\rho)$ and $V_{j-1}(\rho)$ in a standard manner, where j is an iteration index. The results displayed below are for a sufficiently large number of iterations that there is approximate convergence in policy, equilibrium, and value functions.

6.3 Optimal inflation for a new policymaker

If policymaker replacement occurs at the start of a period, then our framework implies that long-term credibility is reset to γ. The strong central bank must then work to rebuild its long-term credibility, but it faces the difficulty that its short-term credibility may be relatively low if there is substantial mimicking by weak policymakers. In Barro [1986], there is a strong policymaker that is constrained to always choose zero inflation and the discretionary policymaker will fully mimic this action if his decision horizon (the length of time until his replacement) is sufficiently lengthy. In our setting, like that of Cukierman and Leviatan [1991], the strong central bank's

[24] Details are provided in appendix A of our working paper version

inflation action is endogenous. Since these prior analyses employ the quasi-linear objective, we also adopt that for this introductory analysis. We thus stress the effect of a strong policymaker's plan on the likelihood, rather than on the intensity, of surprise inflations by the discretionary policymaker.

The evolution of the optimal inflation plan and its consequences are displayed in Figure 4, for a specific set of parameter values shown in Table 3-Case 3 and discussed further in the context of this section. We assume that the discretionary equilibrium inflation rate B is 20 percent per year ($B = .20$). Since $B = \omega_1 \alpha$, we can then specify only one of the objective function weight on the linear output term (ω_1) and the slope of the Lucas-Fischer Phillips curve (α). We are thinking about an annual model, so that a slope of $\alpha = 3$ is broadly consistent with macroeconomic thinking in the 1970s: if output is 3% above trend means that unemployment is 1% below normal (via a form of Okun's law), then $x = 3 * (\pi - e)$ is consistent with a one-for-one short-run trade-off between inflation and unemployment. Accordingly, we require that $\omega_1 = B/\alpha = .2/3 = .067$. We also assume that the strong central bank discounts the future at a rate $b = .96$ (consistent with a four percent annual real interest rate) and that there is a uniform distribution of weak central bank discount factors on $(0, b = .96)$, so that there is a 50% chance that its discount factor falls below one-half that value, so that there is a substantial likelihood of discretionary behavior.

A new central bank is endowed with a stock of long-term credibility of $\gamma = .2$: there is only a 20% chance that it is of the strong type. If it chose an inflation plan of $p = 0$ and also if short-term credibility was equal to long-term credibility, it thus would face expected inflation of $e = \gamma * 0 + (1 - \gamma) * B = .16$ or 16%. That is, the strong central bank inherits an economy that has considerable scepticism about its credibility.

The behavior of the inflation plan (p), expected inflation ($e = \psi p + (1 - \psi)d$) and discretionary inflation $d = B$ are shown in panel A. In the initial period of the new regime, the optimal inflation plan is about 8 per cent per year and expected inflation is about 12 percent per year. The committed central bank works to reduce expected inflation, but does not do so by going "cold turkey" in line with the analysis of Cukierman and Leviatan [1991] because disinflation is costly in real terms. That is: in the initial period, the strong central bank suffers a major output loss with the policy that it does adopt, with output being about 12% below trend and unemployment being about 4% above normal.[25]

Equilibrium mimicking by weak central banks plays an important role in the dynamics of credibility. In the initial period, there is a mimicking rate of about $m = .6$ so that short-term credibility of the policy is much greater than long-term credibility ($\psi = \rho + (1 - \rho)m = .2 + .8 * .6 = .68$). Thus, although private agents see think that it is very likely that the first period of an anti-inflation policy will be carried out, they retain substantial skepticism about the long-term: after the first year of the policy, the level of long-term credibility has risen less to about 30% ($\rho' = \rho/\psi = .2/.68 = .3$) Substantial mimicking is desirable because accumulation of long-term credibility is valuable for weak as well as strong central banks.

With sluggishly growing credibility, there is a lengthy initial recession with the first four years involving output averaging 10% below trend. The optimal inflation policy is quite gradualist, with a inflation decreasing from 8% in the first year to 4% in the fifth year. Inflation expectations are quite stubborn, because there is the prospect of a return to high inflation, but there is ultimately an increase in long-term credibility and short-term credibility, with these measures coming closer together (since $\psi = \rho + (1 - \rho)m$, this is partly accounted for by higher long-term credibility ρ and partly by lower mimicking m).

Overall, inflation dynamics under imperfect information about central bank type are quite different from those of Barro [1986], where there is an exogenous cold-turkey inflation and a long horizon leads to full mimicking by a weak central bank until it becomes sufficiently desirable to burn the credibility in a surprise inflation. The initial behavior of inflation plans are in line with the expressed ideas of Cukierman and Leviatan [1991], although the settings are quite different in one important manner: our policymakers never are known to be near the end of their term of office, so that they do not encounter dramatically decreasing mimicking by weak central banks and the associated rise in private inflation expectations. Cukierman and Leviatan [1991] therefore stress that inflation plans may increase rather than decrease over time.

Our framework allows us to simulate the joint time series behavior of central bank type, inflation actions, real activity, and measures of credibility. While we do not report such simulations here, some key characteristics can be deduced from general features that we have so far discussed. There are occasionally observed changes in central banker: these occur about 16% percent of the time, given that parameter η

[25] $x = \alpha(\pi - e) = 3(.08 - .12) = .12$

is 0.84. Most of the time, the new central banker is a weak one, given that parameter γ is 0.2. With each change of central banker, there is an interval of inflation reduction, which is then followed by a surprise inflation if there is a weak central bank in place. The standard inflation bias equilibrium then prevails ($\pi = B$) until a regime replacement occurs. During the tenure of a strong central bank, there is an initial severe recession, followed by a sustained period of low inflation and normal real activity. However, ultimately, this strong central bank is replaced and the cycle of high inflation and temporary disinflation begins anew. A notable feature of the model is that there is an asymmetry between the periods of low and high output. The length of the intervals of low economic activity is governed by the considerations discussed above if discretionary policy does not take place. But these episodes end in a surprise inflation if a weak central bank has initiated the disinflation. Periods of high economic activity last just for a single period.

6.4 Managing expectations during a transition

What considerations influence the strong central bank's management of expectations during the transition from low credibility? In section 3, with exogenous short-term credibility, we showed that concern about output losses led the strong central bank to partially accommodate inflationary expectations because it realized that these could only be incompletely managed. In this setting, the central bank sought to manage expectations, recognizing that its plan altered the rate of inflation that a weak central bank would choose. In section 4, we made short-term credibility endogenous and showed that the strong central bank sought to manage expectations, recognizing that its plan altered the likelihood of discretionary behavior by a weak central bank.

We now highlight these influences in the dynamic model, where the central bank also recognizes that its plan has an influence on private sector learning and, hence, on future inflation expectations. Concretely, Bayesian learning means that there is more rapid accumulation of long-term credibility if short-term credibility is relatively low, which can be accomplished by adopting an inflation plan that will result in an inflation rate that is unlikely to be produced by a weak central bank. Investment in long-term credibility via means of a low inflation plan, however, comes at a cost in terms of output losses. So, while a long-horizon strong central bank will be more aggressive in terms of its inflation plan than a shorter-term counterpart, it is still the case that it will not adopt a "cold turkey" strategy.

We illustrate these considerations in Figure 5, maintaining B continues to be .20, but introducing a welfare weight on squared output deviations from the first best level (we assume that $\omega_2 = .3$ and requires that ω_1 be cut from .0667 to .0367: we use this parameterization as well in the next subsection). We also adjust the distribution of weak central bank discount factors in order to produce a similar level of optimal inflation plan at $\rho = \gamma = 0.2$ as in the last section with quasi-linear momentary objective function. In particular, the discount factors follows a beta distribution with its shape parameters $(1, 2)$. This parameterization is summarized in case 4 of Table 3.

There are three lines in each of the panels of Figure 5, corresponding to the effects of the inflation plan p at three different levels of credibility ($\rho = .2, .5, .8$). The first 4 panels follow the structure of Figure 2, which represents a static model where the weak type always picks $d = B$ if he chooses to deviate from the inflation plan. In panel A, we see that the mimicking probability responses nonlinearly to the inflation plan, with the nonlinearity more prominent when long-term credibility ρ is lower. These observations contrast sharply with the single linear function $m\,(p)$ in Figure 2 panel A. Such responses are influenced by the inclusion of the quadratic output deviations in the momentary objective function, by the dynamic nature of the strong central bank's maximization, and by the assumed distribution of discount factors.

Panel B of Figure 5 translates the effect of inflation plan on mimicking to short-term credibility via $\psi = \rho + (1 - \rho)m$. Each curve is drawn for a fixed ρ, so the nonlinear responses of m to inflation plan are fully inherited by ψ, at each level of the long-term credibility ρ. Note that a lower inflation plan p decreases its own credibility of being carried out, especially when the long-term credibility ρ is low.

The dynamic model incorporates in the same heterodox effect of the inflation plan on inflation expectations as detailed in section 4: a lower inflation plan dramatically decreases the likelihood of mimicking, so that it may raise the expected inflation $e = \psi p + (1 - \psi)\, d$ over some range at low $\rho = 0.2$, as one can see from panel C. In fact, such an unconventional impact of inflation plan on expectation is even strengthened – the amplification effect of section 4 – due to the optimal reaction of the discretionary inflation rate d displayed in panel E. Using (16), the discretionary inflation rate d assigns more weights on B when the short-term credibility decreases.

There are dramatic output losses, as displayed in panel D, if the inflation plan is very aggressive at a low level of long-term credibility $\rho = 0.2$ because the discretionary inflation rate actually rises with a further decline in the inflation plan. This unconventional reaction of d, together with the heavier weight $(1-\psi)$ assigned to it, make the adverse reaction of expected inflation e to the decline in inflation plan much steeper than the one in Figure 2 panel C.

Finally, panel F shows the gain in long-term credibility next period if the inflation plan is carried out: $\rho' = \rho/\psi$. As discussed in section 6.1, a higher inflation plan increases the mimicking probability and therefore conveys less information about type, resulting in a negative downwards slope of ρ' win response to p. As p approaches B, the likelihood of hitting the inflation plan becomes almost certain regardless of the central bank's type). In this case, the ρ' curves are asymptoting to .2, .5, .8 which are the original levels of long-term credibility ρ.

Notice that the mimicking probability collapses to zero for some range of inflation plan p when $\rho = 0.2$.[26] That is: the strong type is able to prove its toughness immediately if it choose a inflation plan below 3%, gaining full credibility next period as shown in panel F. However, it may not be willing to do so because it would take a deep recession (30% output loss) to get this outcome.

When the optimal inflation plan is formed, it takes into account the output costs that will arise when the plan is executed by the strong central bank. In each panel, the optimal inflation policy is indicated by a "star" at the associated level of long-term credibility. At $\rho = 0.2$, the optimal choice of inflation plan is 6%, which is not low enough to identify the strong type from the weak one. Hence, our model implies that it may not be optimal for a strong central bank to implement a very tough disinflation to gain full credibility, even though it is feasible to do so.

Consideration of optimal policy at all levels of long-term credibility leads to functions capturing how the strong central bank's optimal inflation plan $(p^*(\rho))$, expected inflation $(e(\rho))$, short-term credibility $(\psi(\rho))$, and discretionary inflation $(d(\rho))$ depend on the extent of long-term credibility. Such functions are shown in Figure 6, which contains information relevant for this subsection and the next. In each panel of that Figure, the equilibrium functions are displayed for a "deterministic" case, which is the model described in this subsection and serves as a reference point for our

[26] In the space of (p, ρ) where there is zero mimicking, we come back to the case as detailed in section 3 with $\rho = \psi$. The reaction of e and d in the current setting is consistent with the findings there: decreasing inflation plan lowers both the discretionary inflation rate and the expected inflation, with a greater influence on the latter variable.

discussion of shocks in the next section. The solid lines in the figure thus emphasize that the optimal inflation plan, expected inflation, and discretionary inflation all fall with long-term credibility, while short-term credibility rises. Notice, in particular, that short-term credibility is much higher than long-term credibility, when this is low, reflecting a very high mimicking probability. Such equilibrium functions provide the basis for construction of transition time paths and stochastic simulations of the form discussed in the previous section.

6.5 Oil price shocks and inflation

In the U.S., the temporary oil price shock of 1973 was associated with a substantial and persistent increase in the overall inflation rate, while the major rise of the last few years appears less strongly associated with overall inflation.[27] Ball [1993] develops a model in which there are occasional, large supply shocks: events which substantially reduce the level of real activity when inflation is as expected. In his setting, a weak central bank either mimics the zero inflation behavior of the strong central bank or it partially offsets the supply shock, at a cost of revealing its type and suffering persistently higher inflation as a consequence. Ball [1993] is the first to study a Markov perfect equilibrium with central banks of unknown type, to our knowledge. However, his strong central bank cares only about having zero inflation. In this subsection, we show how our framework can be extended to produce interesting results on the effects of oil price shocks at varying levels of credibility in a setting with a strong central bank that is concerned with both inflation and real activity. Oil price shocks produce a particularly challenging management of expectations, when credibility is at a low level.

6.5.1 Extended framework

We make two modifications of the structure of the prior subsection to more closely match Ball's setting. First, we add an occasional supply shock as in Ball, with

$$x = \alpha(\pi - e) - \varepsilon x^*$$

[27] Interpretation of the second "energy crisis" of 1979 is more complicated, as it was accompanied by a shift in policymaker, with Volcker's policies ultimately dramatically decreasing inflation.

with ε being $\overline{\varepsilon}$ with probability q and 0 with probability $1-q$. Second, we follow Ball in giving the weak monetary authority an objective contains a quadratic inflation term (corresponding to $\omega_2 > 0$, $\omega_1 > 0$ and $\omega_0 = 0$) so that its optimal discretionary action is

$$d = d(e) = \frac{\varphi}{1+\varphi}e + \frac{1}{1+\varphi}B(\varepsilon)$$

with $\varphi = \omega_0 + \omega_2\alpha^2$ and $B(\varepsilon) = \alpha\omega_1 + \alpha\omega_2 x^*(1+\varepsilon)$, as a slight modification of (14). Thus, a weak central bank will choose a higher discretionary inflation rate in face of a supply shock, so long as its objective is not simply linear in real activity ($\omega_2 > 0$). Further, the energy shock ε also has the effect of broadening the range of discount factors over which it is desirable to behave in a discretionary manner (that is, it lowers the likelihood of mimicking).

The introduction of the random supply shock requires that we extend the framework of section 2 to treat ε as a state of the economy. We assume the energy shock is known at the time that all parties take their actions, so that it is relevant to the strong authority's inflation plan ($p(\rho,\varepsilon)$), the private sector's formation of expectations ($e(\rho,\varepsilon)$) and weak central bank's decision to engage in discretionary behavior ($\delta(\rho,\varepsilon)$). However, this is a sufficiently straightforward extension of our recursive approach that it does not require a detailed exposition: one just has a recursive equilibrium with two state variables, ρ and ε. An extension to the analysis of policy response to other supply shocks, such as productivity, is equally direct.

6.5.2 Supply shocks and monetary policy

Under pure discretion, there would be a higher equilibrium inflation rate during the period of an energy shock, since $B(\varepsilon)$ is increasing in ε. At a given plan by the committed central bank, p, the energy shock will produce an increase in discretionary inflation, directly since $d = d(e) = \frac{\varphi}{1+\varphi}e + \frac{1}{1+\varphi}B(\varepsilon)$, and it will produce a decline in mimicking. Hence, short-term credibility will decline. Thus, expected inflation, $e = \psi p + (1-\psi)d(e,\varepsilon)$ will rise at a given p both because discretionary inflation is higher and because more weight is placed on this outcome $(1-\psi)$. Overall, then, a strong central bank confronts a shift in private sector expectations and it must decide how much to accommodate this increase.

First, the strong central bank is concerned about the greater losses in real activity that would prevail if its inflation plan was maintained and expected inflation was higher. Second, the strong central bank knows that the energy price shock will induce a more rapid growth in long-term credibility, other things equal. That is, by hitting a more modest plan when discretionary central banks are less likely to mimic, it will see higher future credibility. In this sense, the energy price move allows the strong central bank a better opportunity to convey information about its type.

This trade-off between current output loss and future credibility gain depends critically on the long-term credibility level ρ possessed by the central bank. Figure 6[28] presents the effect of an energy price shock on a variety of economic variables at all levels of long-term credibility ρ. In each panel, there are three lines: the economy without shocks (solid line), the stochastic economy without an energy shock (dashed line) and the stochastic economy if an energy price shock takes place (dashed-dotted line).

In interpreting the response of various variables, it is important to remember that they are equilibrium outcomes, with the strong central bank selecting an optimal p and the weak central bank choosing the magnitude and likelihood of its discretionary behavior given p. Nevertheless, it is useful to start with a look at how discretionary inflation responds to the energy price shock (in panel A): it is substantially higher, as suggested by the discussion of how a discretionary central bank would behave at a fixed p.

Next, we look at the equilibrium mimicking probability m of the weak central bank in response to the possibility of energy price shock. In panel B, regardless of whether the energy price shock takes place or not, there is significant decline in mimicking relative to the deterministic case. To understand this, it is helpful to recall that the mimicking probability is determined by both the output reward of deviating and the punishment in terms of continuation value. When no shock occurs in current period, the temptation to deviate is the same in stochastic and deterministic setting. The only difference then lies in the future value of obtaining higher credibility, that is, the punishment. In the presence of a potential energy shock that depresses the economy, the expected continuation value declines. Thus, there are smaller losses if the weak central bank decides to burn the credibility now, so that the mimicking probability falls relative to the deterministic case. When the energy price shock hits

[28]In choosing parameters for this experiment, we proceed as follows. First, we allow for a modest quadratic term, $\omega_2 = .3$. Second, we require that the model deliver an inflation bias of B=.20 in the absence of shocks. Given that $B(\varepsilon) = \alpha\omega_1 + \alpha\omega_2 x^*(1+\varepsilon)$, this allows us to determine a value of ω_1 while maintaining the prior values of $\alpha = 3$ and $x^* = .1$. Finally, we use a value of $\bar{\varepsilon} = .1$

the economy, the mimicking probability drops even further, with the extent of the decline increasing with the long-term credibility ρ. In this case, it is the temptation that plays the central role: from panel F, the discrepancy in real activity between commitment and discretionary is widens with greater ρ when there is an energy price shock. However, it is not the case when no shock occurs.

Given the equilibrium behavior of mimicking probability, it is easy to obtain the short-term credibility $\psi = \rho + (1 - \rho)m$ in panel C. At every level of long-term credibility (ρ), short-term credibility declines because mimicking decreases with an energy price shock. The convex shape of short-term credibility in the case of an energy price shock is a direct result from the downward-sloping mimicking probability.

Taking the responses of d and ψ together, it is clear that there will be important effects of the energy price shock ε and the long-term credibility ρ on inflation expectations, because there is more weight being placed on a higher level of discretionary inflation. Panel D bears this out. The expected inflation is higher at lower level of ρ. The energy shock raises the inflation expectations even further, most dramatically for a less credible central bank.[29]

Finally, Panel E shows the strong central bank's optimal reaction to the challenge imposed by increasing inflation expectations associated with the energy price shift. When its credibility is low, the central bank substantially accommodates the expectations shift. However, there is little response if a strong central bank is operating at close to full credibility: it sticks to a low level of inflation in the face of the energy price shock.

Panel F displays the real consequences of the shock and the optimal inflation plan, which are interesting and subtle. When the long-term credibility is at its lowest level ($\gamma = 0.2$), the output losses from carrying out the planned inflation coincide in both cases: the strong central bank sets the inflation plan high enough to avoid any additional recession induced by the energy price shock. Looking back at Figure 5C, we see that expected inflation is essentially unresponsive to the plan near the optimum at such a low level of credibility: a lower plan makes agents very skeptical that it will be carried out. Hence, a lower plan has a big real cost because expectations cannot be managed at all when they increase due to the energy shock. With higher long-term credibility (say, $\rho = .5$), there is a greater degree of leverage on expectations.

[29] Inflation scare effects, in the sense of Goodfriend [1993], are thus most important for low credibile central banks.

So, the strong central bank adopts a tougher plan that involves a larger recession when an energy shock increases expected inflation (there is a greater gap between the dashed-dotted line and the dashed line in Figure 6F). After the long-term credibility ρ accumulates to a sufficient high level, there is no need for the strong central bank to suffer output losses in order to signal his type, because the expected inflation responds little to the shock.[30]

Overall, the mechanisms that we stress are as follows. First, with a supply shock, it is more likely that discretionary monetary policy occurs for the reason stressed by Ball: there are greater rewards to engaging in a temporary stimulation of real activity when there is a supply shock and thus energy price shocks may reveal that the policymaker is of a weak type and lead to a bout of high inflation. Second, energy price shocks confront a strong policymaker with an adverse shift in expectations, which he partly accommodates and partly fights. Both the shift in expected inflation and the strong policymaker's accommodation are smaller when long-term credibility is higher. Finally, the energy price shock allows the strong central bank to build his long-term credibility more rapidly, because he has lower short-term credibility.

7 Summary and conclusions

We provide a basic analysis of expectations management in an environment of imperfect credibility, extending a standard macroeconomic framework for this purpose.[31] A strong central bank pursues low inflation in our framework, but it is also concerned about real activity because it recognizes that its plans are imperfectly credible. In particular, the strong central bank gauges the effect of his policy action on private sector beliefs about the likelihood and intensity of discretionary policy actions that would be taken by an alternative weak central bank.

We consider both a standard rational expectations approach to imperfect information environment, in the spirit of Ramsey analysis of monetary policy, and also a signalling game with strong coherency restrictions on out-of-equilibrium beliefs about

[30] Dotsey and Plosser [2007] discuss this credibility bonus, contrasting the 1970s inflation response to energy shocks and those of recent times.

[31] We distinguish between policy that manages expectations and policy that affects the economy via the coordination of expectations. In other work in progress, Lu and Pasten (2008) explore how fiscal policy can improve economic performance by coordinating expectations. Policy may also affect the incentives that agents have to accumulate information, as in Lu and Pasten (2007). Mertens (2008) provides an alternative approach to analyzing the management of beliefs under discretion.

central bank announcements. In a static setting, we show that these two approaches lead to the same optimal policy announcement by a strong central bank, which will fulfill it definitely, and by a weak central bank, who will not carry it out.

Our analysis also examines optimal expectations management by a strong central bank in a dynamic setting where its announcements and actions affect the evolution of credibility over time. We provide examples of efficient expectations management during a central bank leadership transition with low initial credibility and in response to supply shocks.

Our work leaves open a number of interesting topics, which appear to be feasible applications and extensions of our approach. First, our analysis introduces imperfect information about central bank type only in a very simple manner. It would be of interest to evaluate how the analysis changes if observed policymaker shifts are only noisy signals of underlying changes in regimes. Second, the macroeconomic model that we employ is deliberately "old school" in its inflation dynamics and does not spell out detailed microeconomic foundations. It seems important to explore the nature of inflation management in more realistic macroeconomic models with better micro foundations. Third, issues of expectations management frequently are suggested to be relevant to the choice of the monetary instrument in general and the setting of the chosen instrument at a point in time. By assuming that the central bank directly controls inflation, we have abstained from consideration of these issues, but it seems valuable to do so systematically. Fourth, imperfect credibility and associated topics of expectations management have been suggested to be key to understanding specific historical periods in the monetary histories of the U.S., the U.K., and other countries.[32] It is of interest to begin a more detailed exploration of these connections and episodes. For example, one would like to relate market long-term interest rates to measures of long-term credibility.

More broadly, the relevance of expectations management in environments with limited commitment and imperfect credibility is not limited to inflation and real activity, but seems also central to issues of exchange rate, fiscal and banking policy.

Finally, our approach is to solve a Ramsey-like policy problem presuming a rational expectations equilibrium in a setting of incomplete commitment and imperfect

[32]See, for example, Baxter [1985], Sargent [1986] and Goodfriend and King [2005]. More specifically, our framework can provide a means of answering Baxter's [1988] call for models of imperfectly credible monetary policy to be systematically taken to the data.

information about policymaker type. In a simple setting, it turns out that the predictions of this approach coincide with the unique outcome of a signalling game with strong coherency restrictions on out-of-equilibrium beliefs. It is important to determine the range of environments for which this equivalence holds, since it is a very operational strategy for deducing positive implications of the management of expectations.

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Table 1: Basic Model Ingredients

Panel A: subperiod structure

prior	start	middle	end
transition	plan (p)	expectations (e)	action (π)

Panel B: types and actions

		Type: τ	
		strong	weak
Action: π	Plan	p	p (mimicking)
	Deviation	–	d

Table 2: Long-term credibility, short-term credibility and mimicking

Concept	definition
Long-term credibility	ρ_t =prob($\tau_t = strong$)
Short-term credibility	ψ_t =prob($\pi_t = p_t \vert p_t$)
Mimicking probability	m_t= prob($\pi_t = p_t$)$\vert$ ($p_t, \tau_t = weak$)

Note to Table 2: All probabilities are minimally conditioned on information up to the start of period t. Long-term credibility is the start-of-the period probability that the decision-maker is of the strong type. Short-term credibility is the probability that the inflation plan will actually be carried out, conditional on the plan's level. The mimicking probability is the likelihood, conditional on the decision-maker actually being of the weak type, of inflation being at the planned level.

Table 3: Parameter Values

Parameter	Case 1	Case 2	Case 3	Case 4
x^*	0.1	0.1	0.1	0.1
α	3	3	3	3
γ	-	-	0.2	0.2
η	-	-	0.84	0.84
b	-	-	0.96	0.96
$F(\cdot)$	-	$(\xi/\Xi)^{1/2}$	Uniform	Beta(1,2)
Ξ	-	0.02	-	-
ω_0	0	0	0	0
ω_1	0.0467	0.0667	0.0667	0.03667
ω_2	0.2	0	0	0.3
$\bar{\varepsilon}$	-	-	-	1
q	-	-	-	0.1

Note to Table 3: Case 1 parameter values are used in the introductory Figure 1, discussed in section 3. Case 2 parameter values are used in the static mimicking model of Figure 2, discussed in section 4. Case 3 values are used in the quasilinear dynamic model, discussed in section 6.3. Case 4 values are used in the linear-quadratic dynamic model, discussed in sections 6.4 and 6.5.

Notation Table

Symbol	Definition	section
π	inflation	2
p	inflation plan	2
d	inflation deviation (discretionary inflation)	2
e	expected inflation	2
x	real activity	2
x^*	efficient real activity	2
u	welfare ($u(\pi, x)$)	2
w	reduced form welfare ($w(\pi, e)$)	2
ω_0	welfare weight on unexpected inflation	2
ω_1, ω_2	welfare weight on output terms	2
α	slope of Lucas-Fischer supply function	2
γ	probability of replacing with a strong type	2
η	probability of no replacement	2
ρ	long-term credibility	2
ψ	short-term credibility	2
W	value function for strong central bank	2
V	value function for weak central bank	2
T	temptation	2
P	punishment	2
F	distribution of fixed cost or discount factor	2
β	random discount factor for weak central bank	2
Δ	partial derivative of e wrt π	3
B	inflation bias ($B = \omega_1\alpha + \omega_2\alpha x^*$)	3
θ	reaction coefficient ($\frac{\omega_2\alpha^2\psi}{1+\omega_2\alpha^2\psi}$)	3
ξ	random fixed cost of deviating	4
Ξ	largest fixed cost	4
m	mimicking probability	4
$\tilde{\psi}$	belief of sender's type by observing a signal	5
$h(l)$	sequential equilibrium with higher (lower) signal than p^*	5
q	probability of supply shock	6
ε, $\bar{\varepsilon}$	supply shock, shock size	6

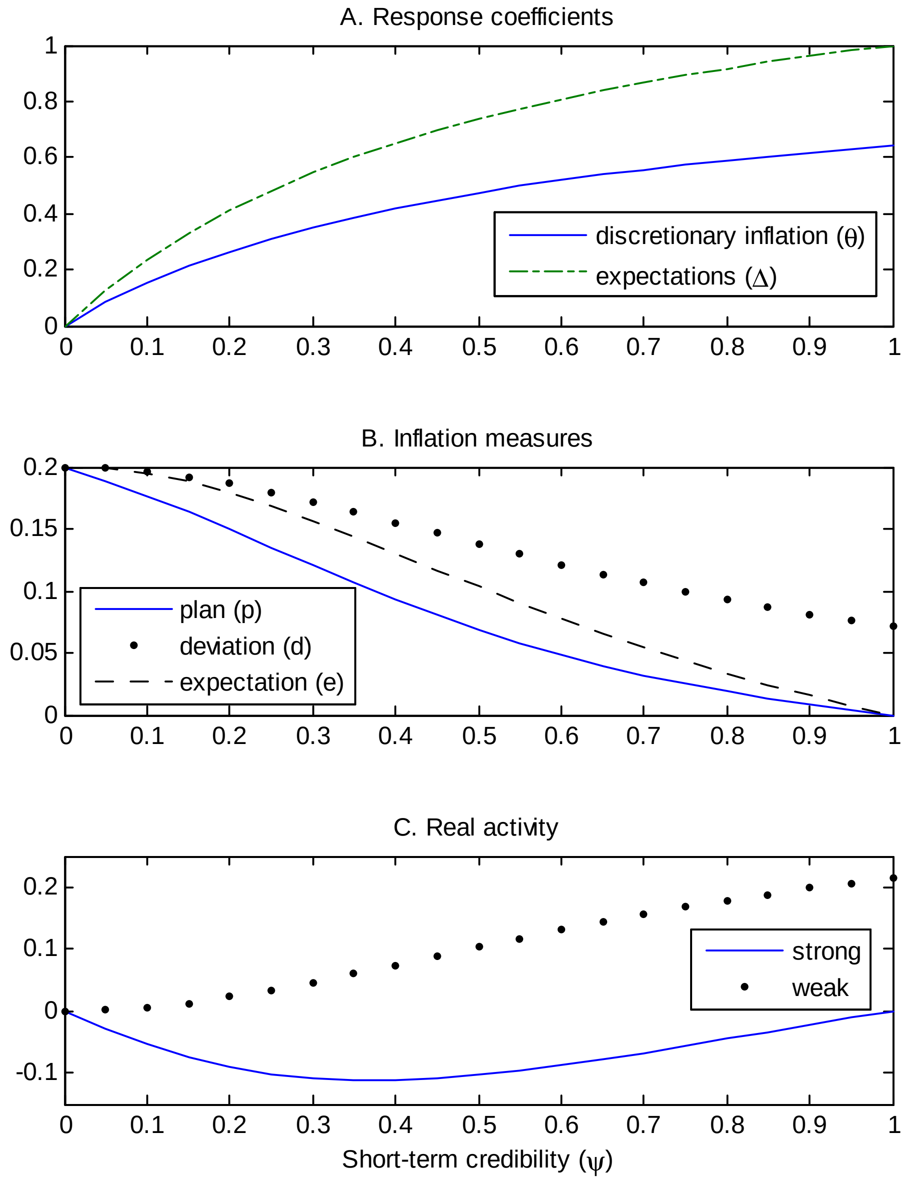

Figure 1: Response to imperfect credibility in static model without mimicking.

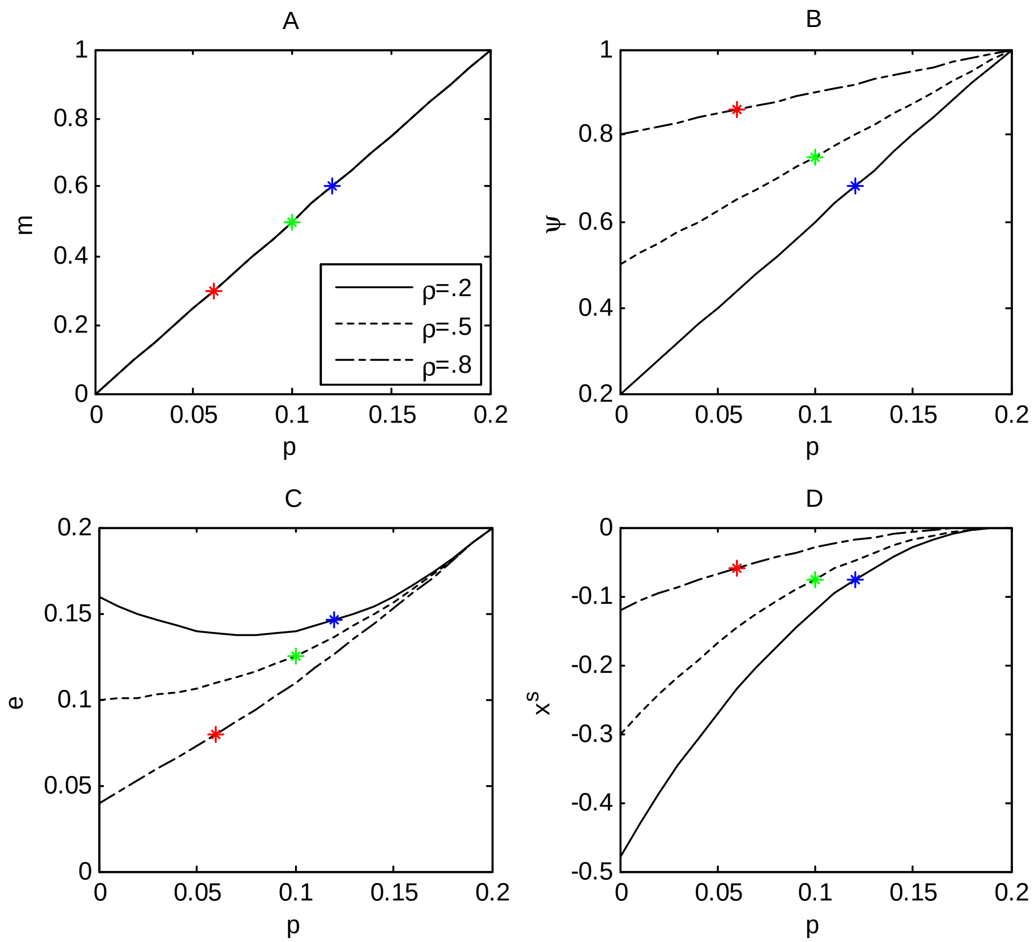

Figure 2: Managing mimicking in static model with quasilinear utility.

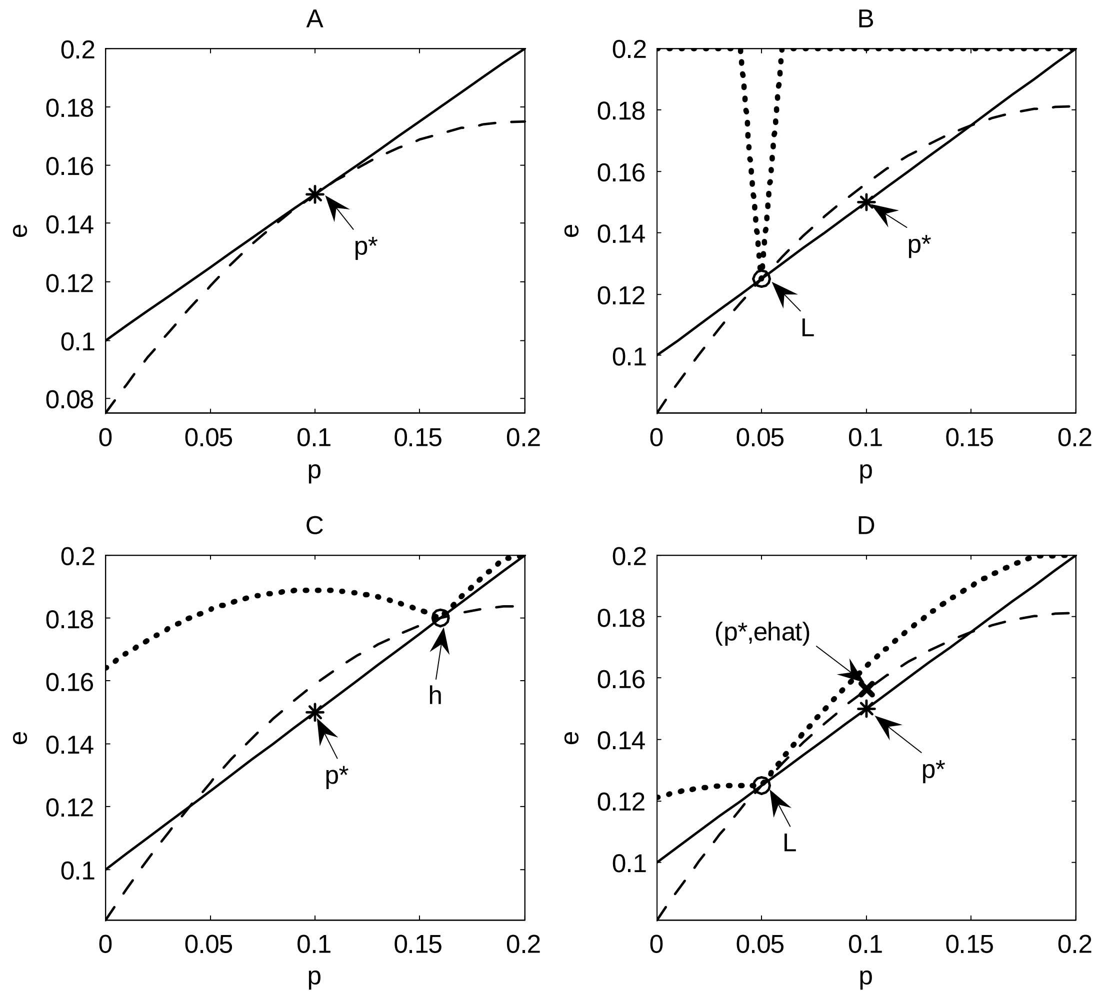

Figure 3: Signalling game

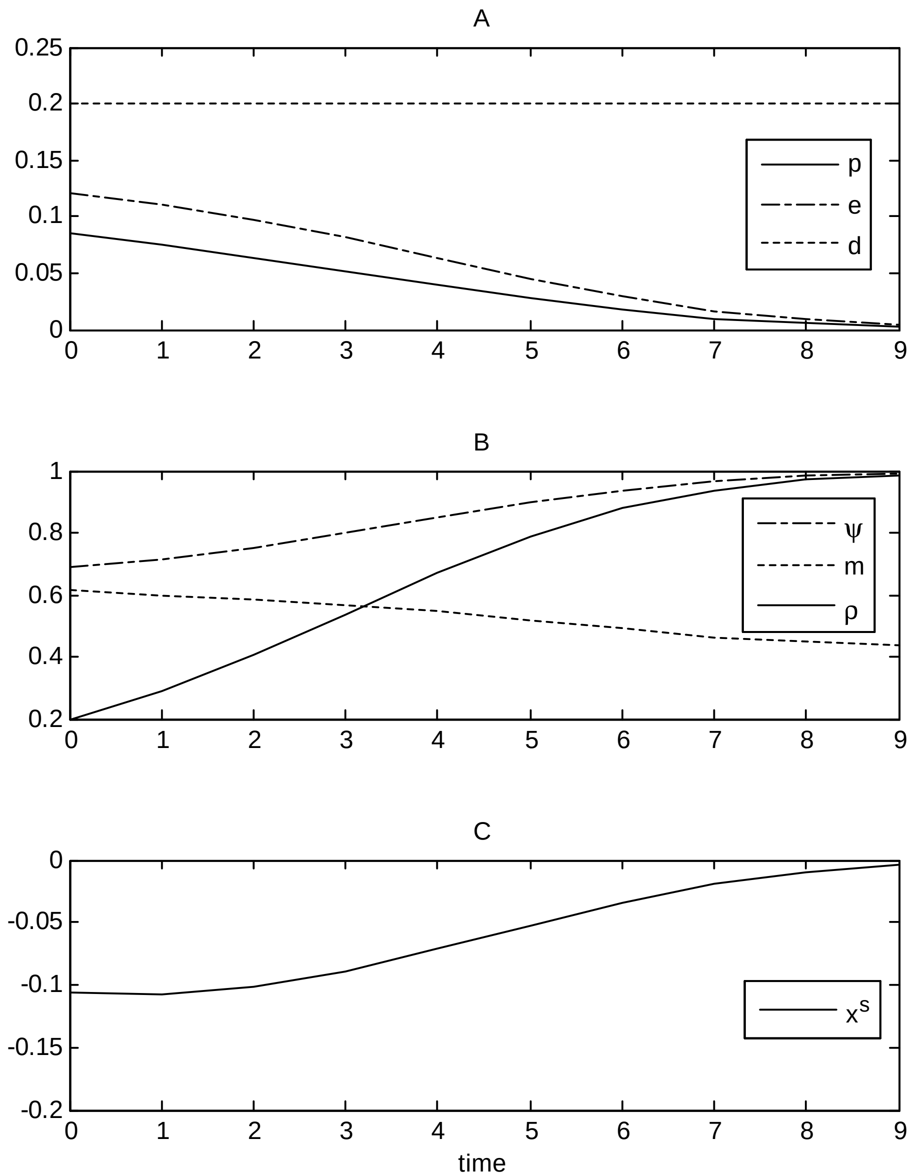

Figure 4: Transition dynamics from low initial credibility ($\gamma = .2$) with quasilinear utility (case 3 parameters).

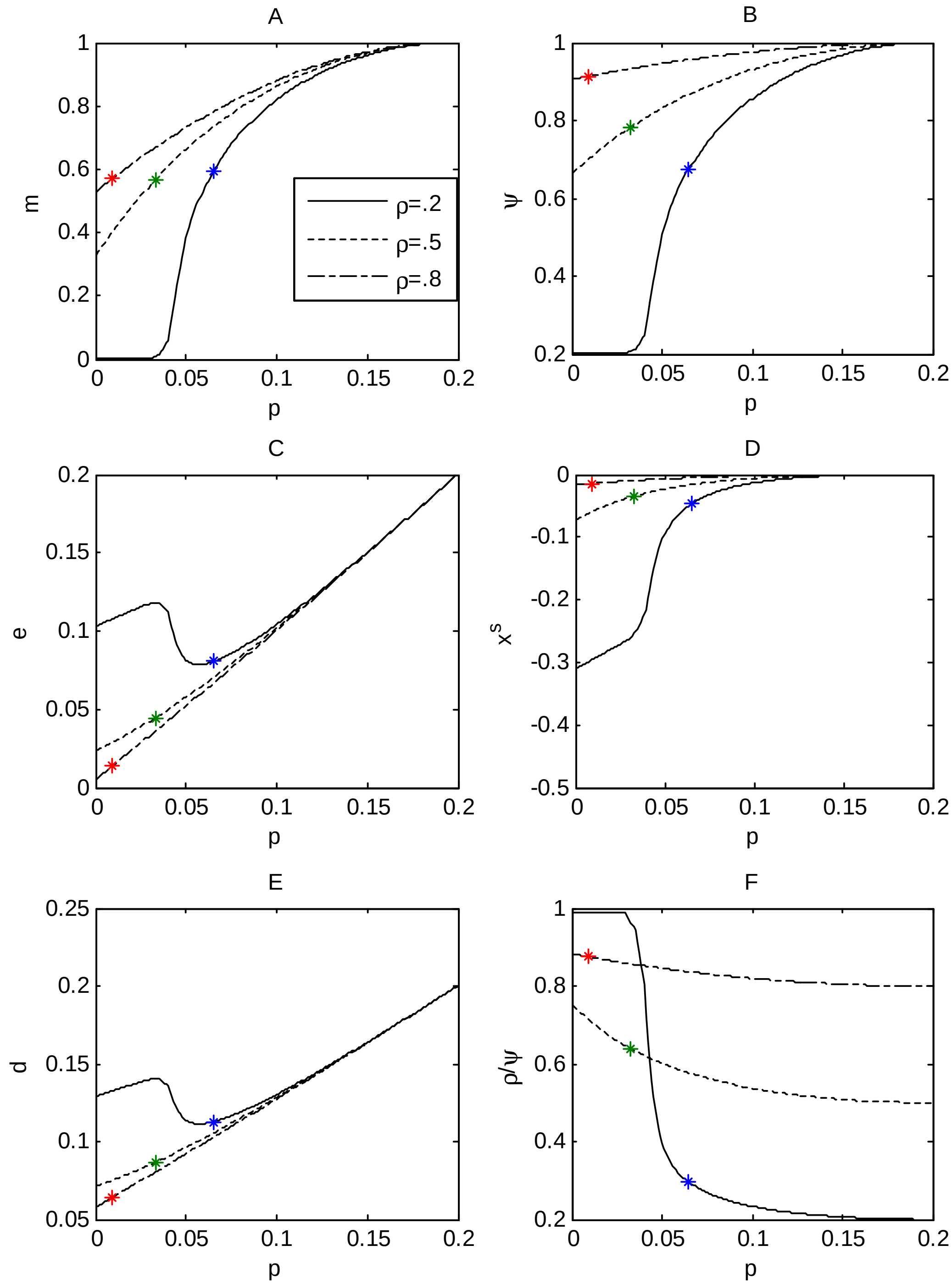

Figure 5: Equilibrium reactions to inflation plan p in full dynamic model (case 4 parameters).

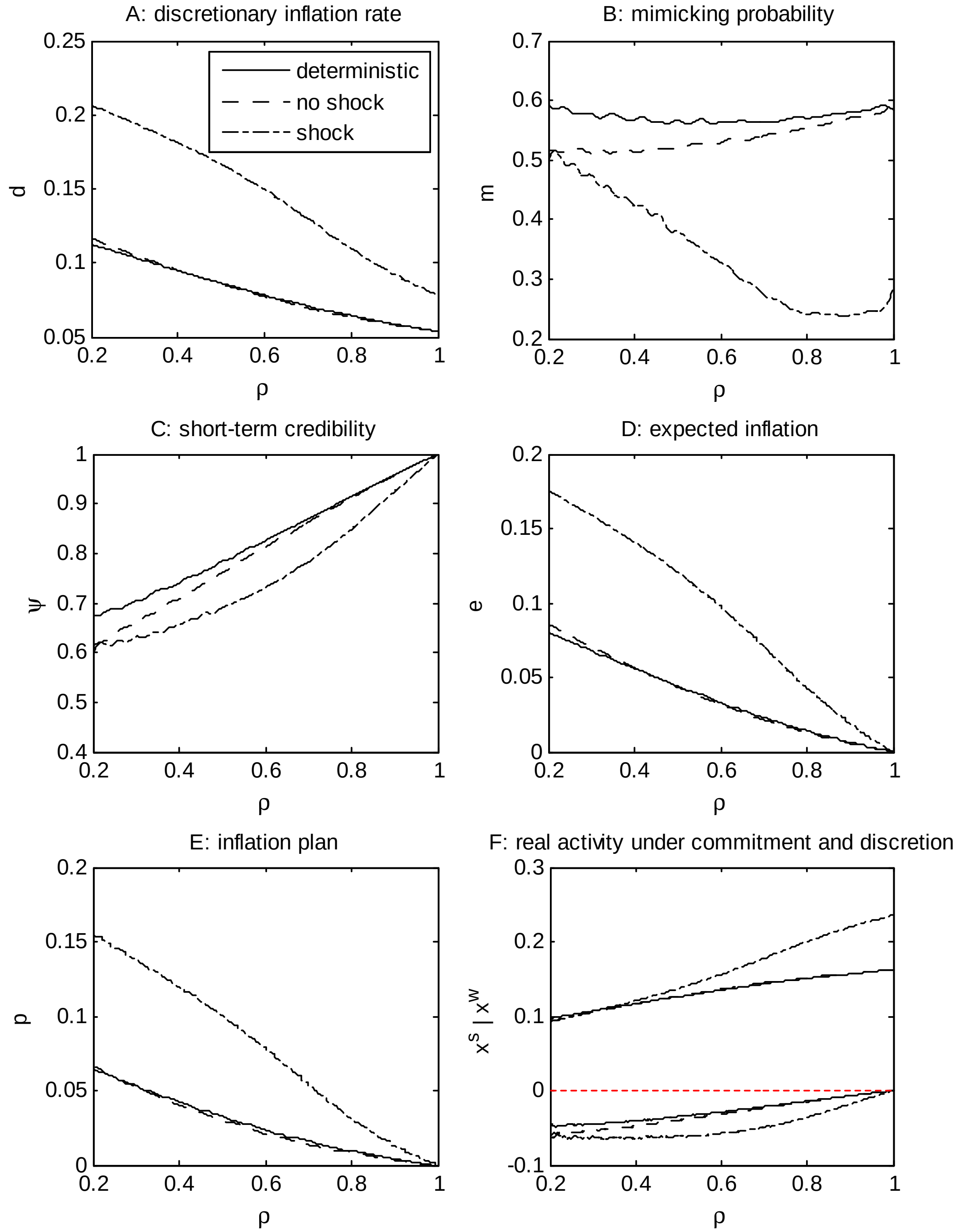

Figure 6: Equilibrium policy functions (case 4 parameters). Solid line is deterministic reference model; dashed-dotted line is stochastic model with shock occurring, while dashed line is when no shock occurs.