Evolving Reputation for Commitment: Understanding US Inflation and Inflation Expectations

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Overview

Managing expectations is central to monetary policy:

- Inflation affected by both expectations and policies
- Past inflation affects expectations
- Expectations respond to CB policy communication

This paper:

- A theory for interaction b/w inflation expectation and policy
- Quantitative theory matches both inflation and expectation well
- Testable implications supported by SPF forecast revision regressions

What we do

- Augment a plain-vanilla NK model with:
  - Private agents learning which policy regime they are in
  - Committed regime policies: managing expectations
  - Opportunistic regime policies: responding to expectations
  - Interplay between agents learning and optimal policies
- New theoretical and numerical approaches:
  - Dynamic game with expectations linkages across periods
  - Mechanism design approach to solve equilibrium
  - Recursive formulation
  - Model-consistent nonlinear Kalman filter with Markov-switching

Our results

- Extract latent states (reputation etc.) only from SPF1Q, SPF3Q
- Model-implied inflation tracks observed inflation
- Policy difference varies with reputation explains Volcker disinflation
- Nonlinear responses of forecast revision to forecast error in SPF consistent with theory

Contribution to the literature

- Learning-based reputation approach: Milgrom and Roberts(1982), Kreps and Wilson(1982), Backus and Driffill(1983), Barro(1986), Phelan(2006), King et al.(2008), Lu(2013), Lu et al.(2016), Dovis and Kirpalani(2021), Morelli an Moretti (2023) etc.

  new approach to solve equilibrium with expectation forward-looking and both types optimizing

- Reputation force as substitute for commitment capability: Barro and Gordon(1983), Chari and Kehoe(1990), Ireland(1997), Kurozumi(2008), Loisel(2008), Sunakawa(2015) etc.

  richer reputation dynamics, punishment varies with deviation from plan

- Literature on US inflation dynamics: Sargent(1999), Primiceri(2006), Bianchi(2013), Matthes(2015), Carvalho et al.(2023), Hazell et al.(2022) etc.

  private sector beliefs and purposeful policymaking jointly determine expected and actual inflation

Policymaker: type and objective

- Committed type (τ_a) chooses and commits to contingent plan $\{a_t\}_{t=0}^{\infty}$
- Opportunistic type (τ_α) chooses intended policy α_t
- Inflation deviates from policy intentions by i.i.d. error $v_\pi \sim N(0, \sigma_{v,\pi})$

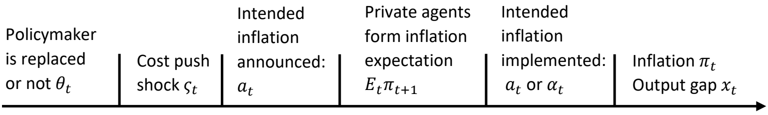
$$\pi_t = \begin{cases} a_t + v_{\pi,t} & \text{with committed type } \tau_a \\ \alpha_t + v_{\pi,t} & \text{with opportunistic type } \tau_\alpha \end{cases} \tag{1}$$

- Quadratic objective in inflation π and output gap x

$$u(\pi, x) = -\frac{1}{2}\{(\pi - \pi^*)^2 + \vartheta_x (x - x^*)^2\} \tag{2}$$

- Committed type (τ_a) patient with β_a
- Opportunistic type (τ_α) myopic with $\beta_\alpha = 0$

Private sector: information and NK inflation dynamics



- Information structure
  - Policymaker is replaced ($\theta = 1$) w/ prob q each period.
  - Replacement event is observed by private agents.
  - Policymaker type and policy intention not observed.
  - Private agents must learn policymaker type from π_t.

- NK standard Phillips curve

$$\pi_t = \underbrace{\beta E_t^p \pi_{t+1}}_{e_t} + \kappa x_t + \varsigma_t \tag{3}$$

ς Markov-chain cost-push shock

Reputation and Inflation Expectations

- History within a regime $h_t = \{h_{t-1}, \pi_{t-1}, \varsigma_t\}$
- **Reputation** within a regime $\rho(h_t) = \Pr(\tau_a | h_t)$

$$\rho(h_{t+1}) = \rho(h_t, \pi_t) \equiv \frac{\rho(h_t)\, g(\pi_t | a(h_t))}{\rho(h_t)\, g(\pi_t | a(h_t)) + (1 - \rho(h_t)) g(\pi_t | \alpha(h_t))} \quad (4)$$

- Private sector inflation expectations: Detail

$$\begin{aligned} e(h_t) &= \beta E^p(\pi_{t+1} | h_t) \\ &= \beta \rho(h_t) \underbrace{E\pi_{t+1} | (h_t, \tau_a)}_{\text{committed policy}} + \beta(1 - \rho(h_t)) \underbrace{E\pi_{t+1} | (h_t, \tau_\alpha)}_{\text{opportunistic policy}} \end{aligned} \quad (5)$$

- Reputation passes on to a new regime with prob δ_ρ
  - New policymaker's reputation $\rho_0 = \phi_t \rho(h_t) + (1 - \phi_t) v_{\rho,t}$
  - $\phi_t \sim$ Bernoulli(δ_ρ) and $v_{\rho,t} \sim$ Beta$(\overline{\rho}, \sigma_\rho)$.

Optimal opportunistic policy: myopic

- Opportunistic type chooses α_t that generates $\pi_t = \alpha_t + v_{\pi,t}$

$$\alpha_t = \underset{\alpha_t}{\operatorname{argmax}} \int u(\pi_t, \frac{\pi_t - e_t - \varsigma_t}{\kappa}) g\left(\pi_t | \alpha_t\right) d\pi_t \tag{6}$$

  taking $e_t = e(h_t)$ as given

- Linear best response

$$\alpha(h_t) = Ae(h_t) + B(\varsigma_t) \tag{7}$$

Forward-looking alternative

Inflation bias varies with expectation

contrasting two concepts

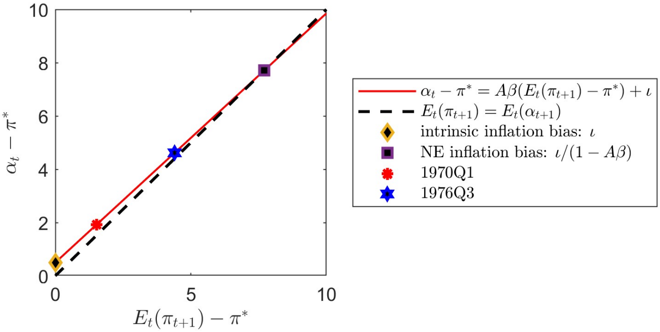
$$\alpha(e) = Ae + B(\varsigma), A = .94, \beta = .995$$

- Intrinsic inflation bias (small)

$$\alpha(e = \beta\pi^*) - \pi^* = 0.5\%.$$

- Nash Eq inflation bias (BIG)

$$\alpha(e = \beta\alpha) - \pi^* = 8\%$$



Optimal committed policy plan

At start of his term, choose $\{a(h_t)\}_{t=0}^{\infty}$ to maximize

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta_a^t (1-q)^t \underline{u}\left(a(h_t), e(h_t), \varsigma_t\right)$$

$\underline{u}\left(a, e, \varsigma\right) \equiv \int u(\pi, x(\pi, e, \varsigma)) g(\pi | a) d\pi$ and $x(\pi, e, \varsigma) = \frac{\pi - e - \varsigma}{\kappa}$

- "Strategic power" of $\{a(h_t)\}_{t=0}^{\infty}$ on $\{e(h_t)\}_{t=0}^{\infty}$
  - **anchor expectation**: $a(h_{t+1})$ directly affects $e(h_t)$
  - **manage perceived alternative**: $\alpha(h_t)$ best response to $e(h_t)$
  - **build reputation**: $a(h_{t-1})$ and $\alpha(h_{t-1})$ affect $\rho(h_t)$.

Mechanism design approach for within-regime equilibrium

Committed type chooses $\{a_t, \alpha_t, e_t\}_{t=0}^{\infty}$ to maximize

$$U_0 = E_0\{\sum_{t=0}^{\infty} \beta_a^t (1-q)^t \, \underline{u}(a_t, e_t, \varsigma_t)\} \tag{8}$$

subject to 3 constraints each period:

1. Rational inflation expectations: $e_t = \beta E_t^p \pi_{t+1}$
2. Incentive compatibility of opportunistic policy: $\alpha_t = Ae_t + B(\varsigma_t)$
3. Bayesian learning: $\rho_{t+1} = \frac{\rho_t g(\pi_t|a_t)}{\rho_t g(\pi_t|a_t)+(1-\rho_t)g(\pi_t|\alpha_t)}$

Change of measure

Recursive optimal policy problem for committed type

Generalization of Bellman (using pseudo state μ):

$$\begin{aligned} W(\varsigma, \rho, \mu) &= \min_{\gamma} \max_{a,\alpha,e} \{ \underline{u}(a, e, \varsigma, \tau_a) + (\gamma e + \mu \omega) \\ &\quad + \beta_a (1-q) \int \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) W(\varsigma', \rho', \mu') g(\pi | a) d\pi \} \end{aligned} \tag{9}$$

subject to $\alpha = Ae + B(\varsigma)$ and

$$\omega \equiv -[(1-q)a + qz] - \frac{1-\rho}{\rho}[(1-q)\alpha + qz] \tag{10}$$

$$\mu' = \frac{\beta}{\beta_a(1-q)} \gamma \rho, \text{ with } \mu_0 = 0 \tag{11}$$

$$\rho' = \frac{\rho g(\pi | a)}{\rho g(\pi | a) + (1-\rho) g(\pi | \alpha)} \tag{12}$$

Contrast to NK Ramsey

Linking the theory to the data

Model inputs

- 3 structural shocks $v_t = (v_\varsigma, v_\rho, v_\pi)$
- 3 state variables $s_t = (\varsigma_t, \rho_t, \mu_t)$
- 3 discrete states $\Theta_t = (\theta_t, \phi_t, \tau_t)$ Def and Trans

Model outputs:

- committed and opportunistic policies $a\,(s_t)$ and $\alpha\,(s_t)$
- inflation $\pi_t = \tau_t a(s_t) + (1 - \tau_t)\alpha(s_t) + v_{\pi,t}$
- inflation forecasts at various horizons $E^p(\pi_{t+k}|s_t) = e\,(s_t, k)$

Data:

- SPF inflation forecasts at various horizons
- Inflation, food and energy price shock

State space model with Markov-switching

$$X_t = [\varsigma_t, \rho_t, \mu_t, \pi_t]' = F(X_{t-1}, v_t|\Theta_t = (\theta_t, \phi_t, \tau_t))$$

$$= \left[\begin{array}{c} \delta_\varsigma \varsigma_{t-1} + v_{\varsigma,t} \\ (1-\theta_t+\theta_t\phi_t)b(\varsigma_{t-1},\rho_{t-1},\mu_{t-1},\pi_{t-1}) + \theta_t(1-\phi_t)v_{\rho,t} \\ (1-\theta_t)m(\varsigma_{t-1},\rho_{t-1},\mu_{t-1}) \\ \tau_t a(\varsigma_t,\rho_t,\mu_t) + (1-\tau_t)\alpha(\varsigma_t,\rho_t,\mu_t) + v_{\pi,t} \end{array} \right]$$

$$Y_t = \left[\begin{array}{c} f_{t+1|t} \\ f_{t+2|t} \\ f_{t+3|t} \\ f_{t+4|t} \\ \frac{1}{40}\sum_{k=1}^{40} f_{t+k|t} \\ \widetilde{\pi}_t \\ \widetilde{\varsigma}_t \end{array} \right] = \left[\begin{array}{c} e(\varsigma_t,\rho_t,\mu_t,1) + u_{1t} \\ e(\varsigma_t,\rho_t,\mu_t,2) + u_{2t} \\ e(\varsigma_t,\rho_t,\mu_t,3) + u_{3t} \\ e(\varsigma_t,\rho_t,\mu_t,4) + u_{4t} \\ \overline{e}(\varsigma_t,\rho_t,\mu_t,40) + u_{40,t} \\ \pi_t + u_{\pi t} \\ \varsigma_t + u_{zt} \end{array} \right] = H(X_t, u_t)$$

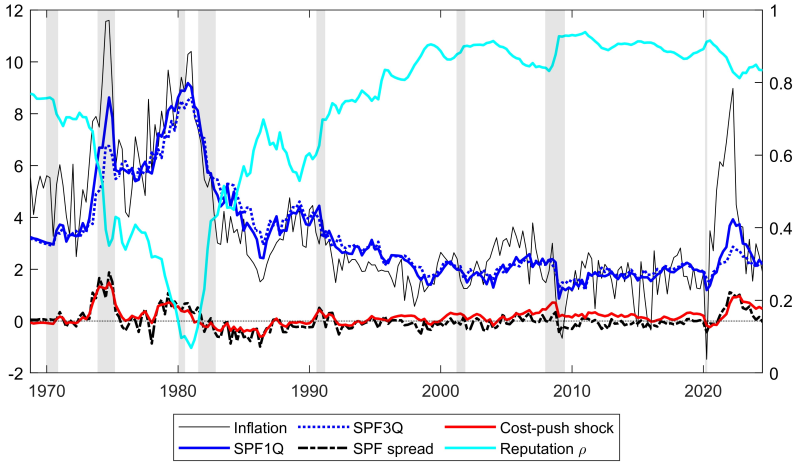
State space model with Markov-switching

$$X_t = [\varsigma_t, \rho_t, \mu_t, \pi_t]' = F(X_{t-1}, v_t | \Theta_t = (\theta_t, \phi_t, \tau_t))$$

$$= \begin{bmatrix} \delta_\varsigma \varsigma_{t-1} + v_{\varsigma,t} \\ (1-\theta_t+\theta_t\phi_t) b(\varsigma_{t-1}, \rho_{t-1}, \mu_{t-1}, \pi_{t-1}) + \theta_t(1-\phi_t) v_{\rho,t} \\ (1-\theta_t) m(\varsigma_{t-1}, \rho_{t-1}, \mu_{t-1}) \\ \tau_t a(\varsigma_t, \rho_t, \mu_t) + (1-\tau_t)\alpha(\varsigma_t, \rho_t, \mu_t) + v_{\pi,t} \end{bmatrix}$$

$$Y_t = \begin{bmatrix} f_{t+1|t} \\ f_{t+2|t} \\ f_{t+3|t} \\ f_{t+4|t} \\ \frac{1}{40}\sum_{k=1}^{40} f_{t+k|t} \\ \widetilde{\pi}_t \\ \widetilde{\varsigma}_t \end{bmatrix} = \begin{bmatrix} e(\varsigma_t, \rho_t, \mu_t, 1) + u_{1t} \\ e(\varsigma_t, \rho_t, \mu_t, 2) + u_{2t} \\ e(\varsigma_t, \rho_t, \mu_t, 3) + u_{3t} \\ e(\varsigma_t, \rho_t, \mu_t, 4) + u_{4t} \\ \overline{e}(\varsigma_t, \rho_t, \mu_t, 40) + u_{40,t} \\ \pi_t + u_{\pi t} \\ \varsigma_t + u_{zt} \end{bmatrix} = H(X_t, u_t)$$

Extracting states: term structure intuition about SPF



- SPF1Q more sensitive to temporary price shocks
- SPF3Q better reflects reputation

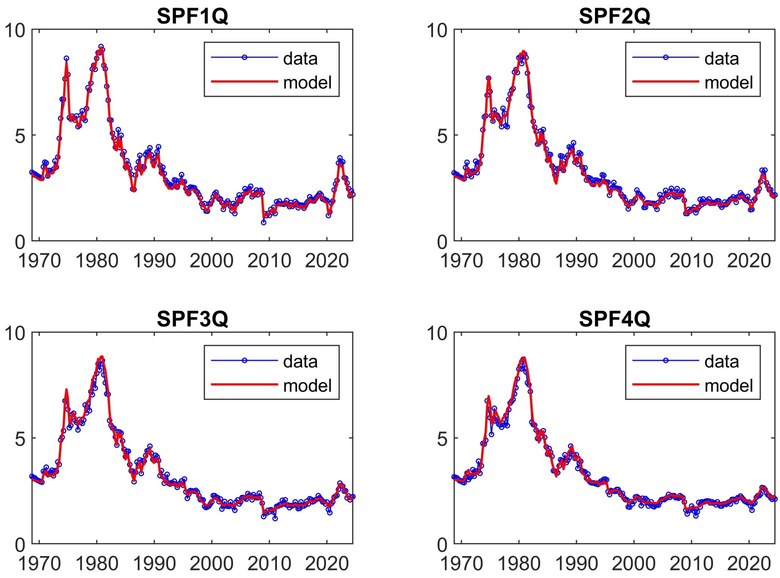
Calibration of parameters

β, β_a	Discount factor (private, committed type)	0.995
q	Replacement probability	0.03
κ	PC output slope	0.08
π^*	Inflation target	1.5%
ϑ_x	Output weight	0.1
x^*	Output target	1.73%
δ_ς	Persistence of cost-push shock	0.7
$\sigma_{v,\varsigma}$	Std of cost-push innovation	0.7%
$\sigma_{v,\pi}$	Std of implementation error v_π	1.2%
δ_ρ	prob of reputation inheritance	0.9
$\overline{\rho}$	mean of reputation draw	0.1
σ_ρ	std of reputation draw	0.05

- Implies $A = 0.94$, $\iota = 0.5\%$, NE bias$= 8\%$

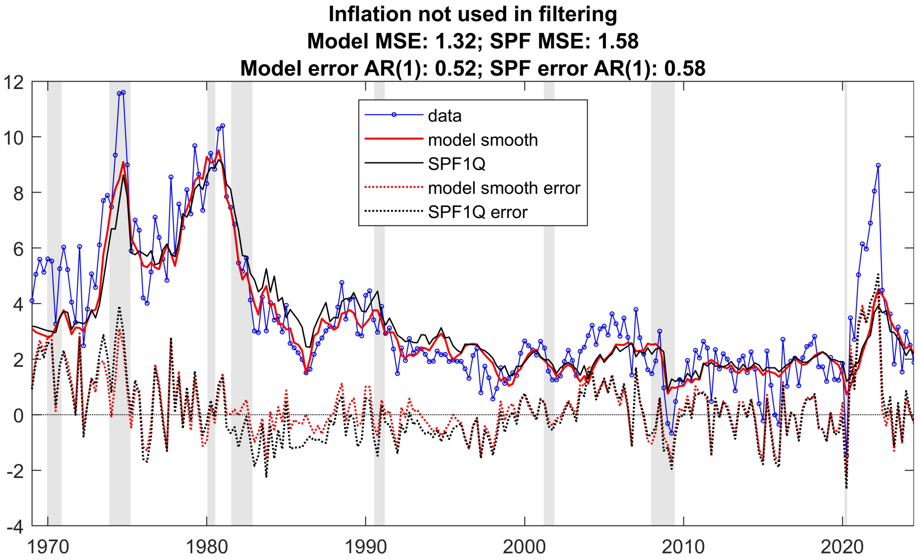
Calibration

SPFs: targeted and untargeted



Untargeted: Inflation

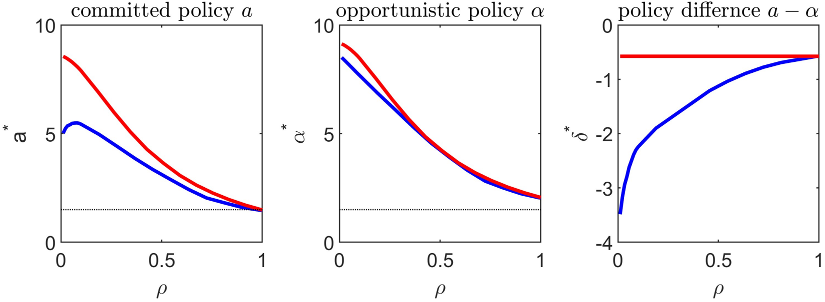
filtered result



Intentional Reputation Management

Relative to literature with learning/regime change:

- Our **committed type influences private agents' belief** (reputation)
  - larger policy difference at lower ρ
- Naive committed type treats reputation as exogenous
  - policy difference independent of ρ

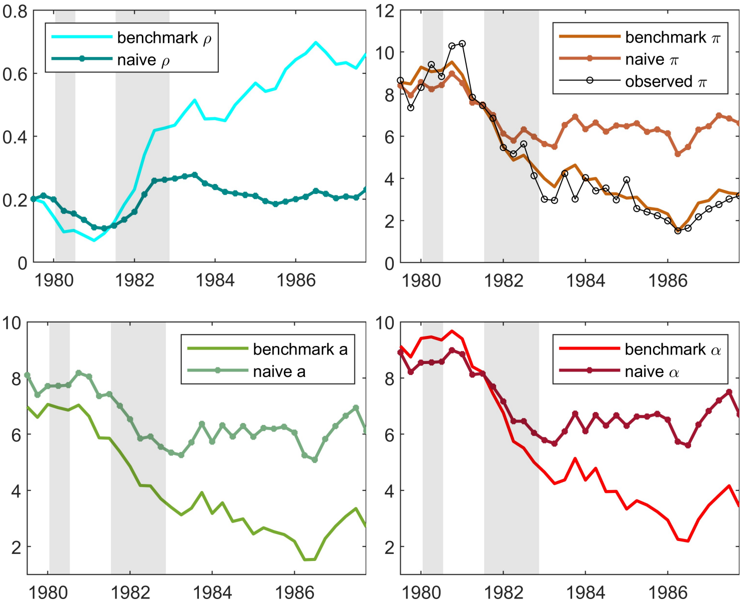


Counterfactuals with Naive Committed Policy

Committed policymaker treats reputation as an exogenous process:

- reputation still evolves endogenously according to Bayes' rule
- CB responds to time-varying reputation (Kreps, Cogley & Sargent)
- same cost-push shocks, discrete states prob, implementation errors
- focus on Volcker period: committed, low reputation

Volcker Disinflation



Understanding long-term inflation expectations

Our theory implies reputation process:

$$\rho_{t+1} - \rho_t \approx k\underbrace{\{\rho_t(1-\rho_t)[a_t - \alpha_t]\}}_{\text{time-varying coeff}}\underbrace{\{\pi_t - [\rho_t a_t + (1-\rho_t)\alpha_t]\}}_{\text{now-cast error}} \tag{13}$$

- ρ determines long-term inflation expectation

  $f_{\infty|t} = (1-q)[\rho_t \pi^* + (1-\rho_t)\pi^{NE}] + q[\rho_t z_{\varsigma=0,\rho=1} + (1-\rho_t) z_{\varsigma=0,\rho=0}]$

  10-y CPI forecast $E_t \pi_{t+40} \Rightarrow \widehat{\rho}_t$
- $|a - \alpha|$ larger at lower ρ
- Time-varying coefficient increasing in $\rho(1-\rho)^2$

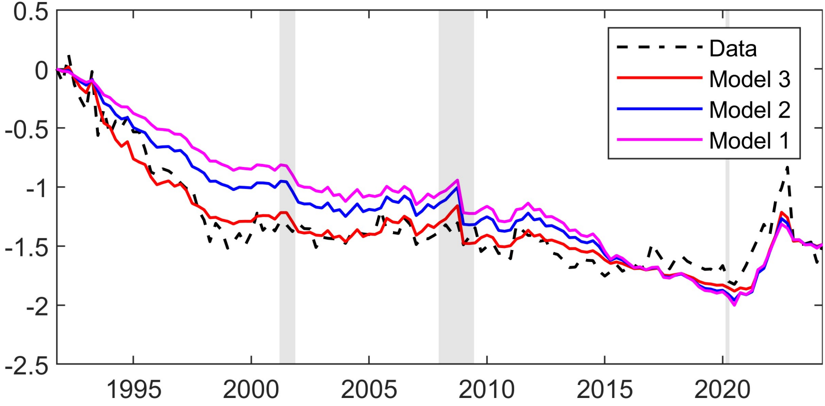
Forecast revision regression

$$E_{t+1}\pi_{t+40} - E_t\pi_{t+40} = \gamma_t(\pi_t - E_t\pi_t) + \varepsilon_t \tag{14}$$

- Model 1: $\gamma_t = \gamma$
- Model 2: $\gamma_t = \gamma\widehat{\rho}_t(1-\widehat{\rho}_t)$
- Model 3: $\gamma_t = \gamma\widehat{\rho}_t(1-\widehat{\rho}_t)^2$

	γ	p value	Adjusted R^2	RMSE	N
Model 1	0.038	1.13E-06	0.163	0.117	130
Model 2	0.384	1.04E-07	0.193	0.114	130
Model 3	3.078	1.02E-09	0.248	0.11	130

Forecast revision regression: cumulative change

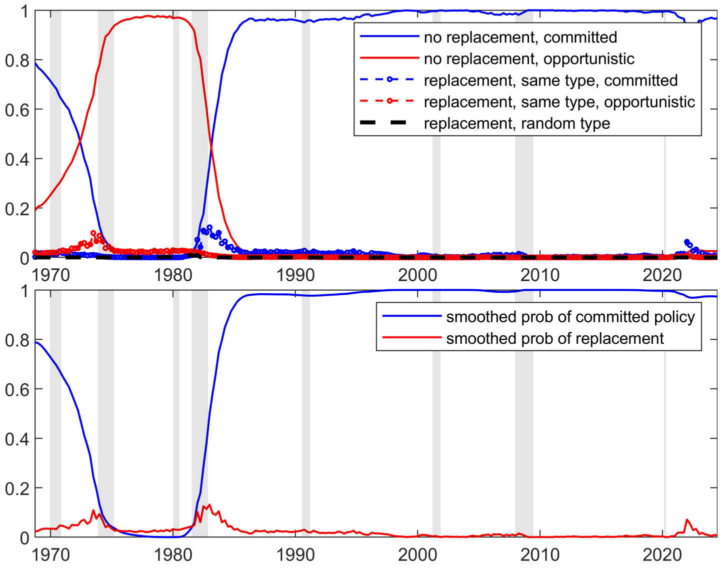


Cumulative change in long-term CPI inflation forecast 1991Q4-2024Q2

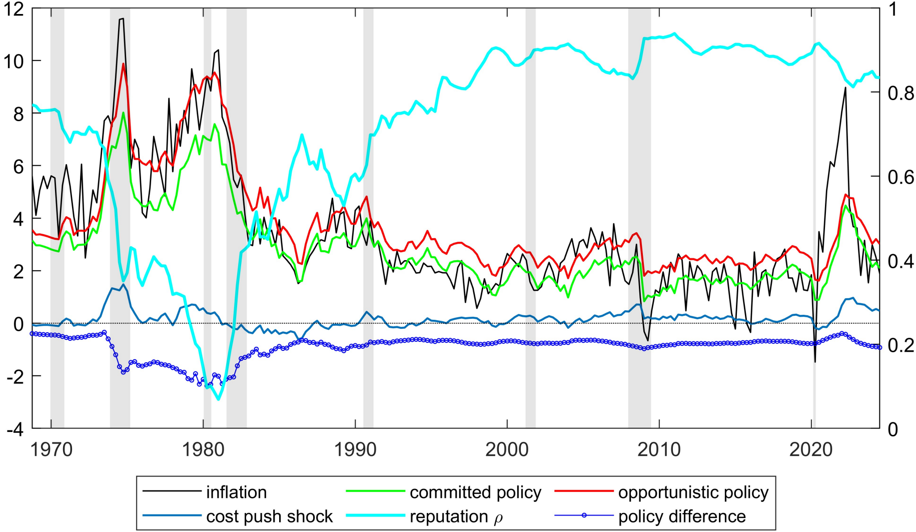
Summary and conclusions

- A theory for interaction b/w inflation expectation and policy
  - Private agents learns type and form expectations of future policy
  - Committed policymaker manages expectations
  - Opportunistic policymaker responds to expectations
  - Interplay between agents learning and optimal policies
- Quantitative theory matches both inflation and expectation well
- Testable implications supported by SPF forecast revision regressions

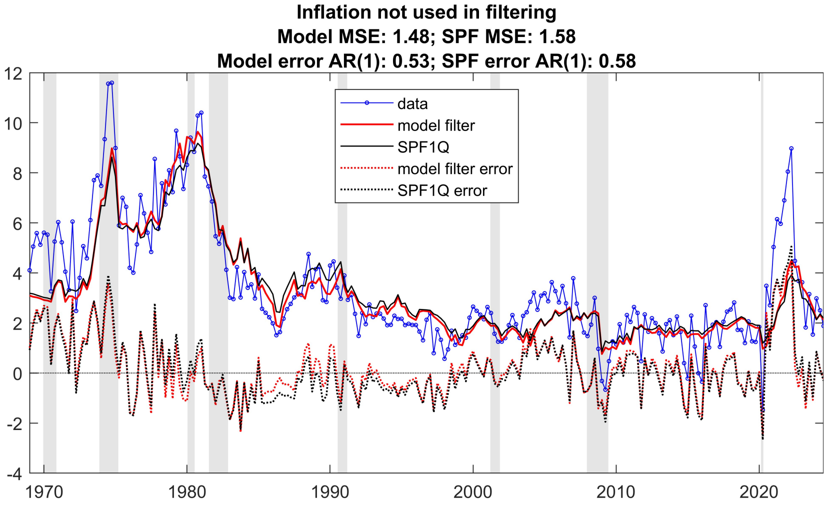
Smoothed Probability



Model-based interpretation of inflation history



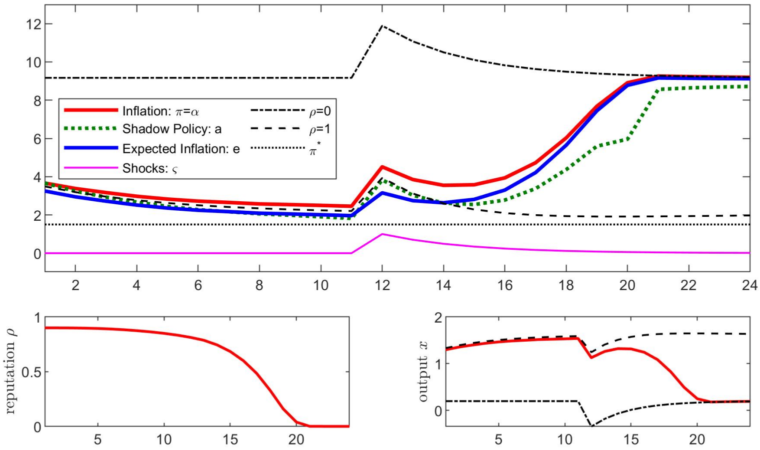
Untargeted inflation with filtered results return



Opportunistic regime simulation

Great inflation style: high initial reputation 0.9, response to 1% supply shock in t=12

$\pi = \alpha$: slow learning for a long while, supply shock speeds up learning



Optimal opportunistic policy: forward-looking

Opportunistic type chooses α_t that generates $\pi_t = \alpha_t + v_{\pi,t}$

- takes e_t as given but ... understands:
- future payoff depends on future expected inflation $e(h_{t+1})$
- $e(h_{t+1})$ depends on current inflation $h_{t+1} = \{h_t, \pi_t, \varsigma_{t+1}\}$
- manages $e(h_{t+1})$ in a limited manner by controlling π_t

$\alpha_t := \alpha(h_t)$ is sequentially rational if it satisfies the first-order condition

$$\begin{aligned} 0 &= \int u(\pi_t, e_t, \varsigma_t) \frac{\partial g\left(\pi_t | \alpha_t\right)}{\partial \alpha_t} d\pi_t \\ &\quad + \beta_\alpha (1-q) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) V\left(h_t, \pi_t, \varsigma_{t+1}\right) \frac{\partial g\left(\pi_t | \alpha_t\right)}{\partial \alpha_t} d\pi_t \end{aligned} \tag{15}$$

with

$$\begin{aligned} V(h_t) &= \int u(\pi_t, e_t, \varsigma_t) g\left(\pi_t | \alpha_t\right) d\pi_t \\ &\quad + \beta_\alpha (1-q) \int \sum_{\varsigma_{t+1}} \varphi(\varsigma_{t+1}; \varsigma_t) V\left(h_t, \pi_t, \varsigma_{t+1}\right) g\left(\pi_t | \alpha_t\right) d\pi_t \end{aligned} \tag{16}$$

Optimal committed policy plan

At start of his term, choose $\{a(h_t)\}_{t=0}^{\infty}$ to maximize

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta_a^t (1-q)^t \underline{u}\left(a(h_t), e(h_t), \varsigma_t\right)$$

where $\underline{u}\left(a, e, \varsigma\right) \equiv \int u(\pi, x(\pi, e, \varsigma)) g(\pi|a) d\pi$

- “Strategic power" of $\{a(h_t)\}_{t=0}^{\infty}$ on $\{e(h_t)\}_{t=0}^{\infty}$
  - **anchor expectation**: $e(h_t)$ anchored by $\rho(h_t)a(h_{t+1})$
  - **manage perceived alternative**: $\alpha(h_t)$ affected by $e(h_t)$ and $e(h_{t+1})$
  - **build reputation**: $\rho(h_t)$ affected by $a(h_{t-1})$ and $\alpha(h_{t-1})$

Mechanism design approach for within-regime equilibrium

Committed type chooses $\{a_t, \alpha_t, e_t\}_{t=0}^{\infty}$ to maximize

$$U_0 = E_0\{\sum_{t=0}^{\infty} \beta_a^t (1-q)^t \underline{u}(a_t, e_t, \varsigma_t)\} \tag{17}$$

subject to 3 constraints each period:

1. Rational inflation expectations for private agents

$$e_t = \beta \int \sum \varphi(\varsigma_{t+1}; \varsigma_t)\{\rho_t[(1-q)a_{t+1} + qz_{t+1}]g(\pi_t|a_t) \\ +(1-\rho_t)[(1-q)\alpha_{t+1} + qz_{t+1}]g(\pi_t|\alpha_t)\}d\pi_t$$

2. Sequential rationality conditions for opportunistic type

$$0 = \frac{\partial \underline{u}(\alpha_t, e_t, \varsigma_t)}{\partial \alpha_t} + \beta_\alpha(1-q)\int \sum \varphi(\varsigma_{t+1}; \varsigma_t) V_{t+1} \frac{\partial g(\pi_t|\alpha_t)}{\partial \alpha_t} d\pi_t$$

$$V_t = \underline{u}(\alpha_t, e_t, \varsigma_t) + \beta_\alpha(1-q)\int \sum \varphi(\varsigma_{t+1}; \varsigma_t) V_{t+1} g(\pi_t|\alpha_t)\, d\pi_t$$

Recursive formulation (Marcet and Marimon 2019)

Committed type chooses $\{a_t, \alpha_t, e_t\}_{t=0}^{\infty}$ to maximize

$$U_0 = E_0\{\sum_{t=0}^{\infty} \beta_a^t (1-q)^t \underline{u}(a_t, e_t, \varsigma_t)\} \quad (18)$$

subject to 3 constraints each period:

1. Rational inflation expectations for private agents

$$\gamma_t : e_t = \beta \int \sum \varphi(\varsigma_{t+1}; \varsigma_t)\{\rho_t[(1-q)a_{t+1} + qz_{t+1}]g(\pi_t|a_t) \\ +(1-\rho_t)[(1-q)\alpha_{t+1} + qz_{t+1}]g(\pi_t|\alpha_t)\}d\pi_t$$

2. Sequential rationality conditions for opportunistic type

$$\phi_t : 0 = \frac{\partial \underline{u}(\alpha_t, e_t, \varsigma_t)}{\partial \alpha_t} + \beta_\alpha(1-q)\int \sum \varphi(\varsigma_{t+1}; \varsigma_t)V_{t+1}\frac{\partial g(\pi_t|\alpha_t)}{\partial \alpha_t}d\pi_t$$

$$\chi_t : V_t = \underline{u}(\alpha_t, e_t, \varsigma_t) + \beta_\alpha(1-q)\int \sum \varphi(\varsigma_{t+1}; \varsigma_t)V_{t+1}g(\pi_t|\alpha_t)\, d\pi_t$$

Change of measure

Recursive formulation (Marcet and Marimon 2019)

Within-regime equilibrium is the solution to

$$W(\varsigma, \rho, \mu, y) = \min_{\gamma,\phi,\chi} \max_{a,\alpha,e,V} \underline{u}(a, e, \varsigma) + (\gamma e - \mu\omega) \tag{19}$$

$$+ \phi \frac{\partial \underline{u}(\alpha, e, \varsigma)}{\partial \alpha} + \chi \underline{u}(\alpha, e, \varsigma) + (y - \chi)V$$

$$+ \beta_a(1-q) \int \sum_{\varsigma'} \varphi(\varsigma'; \varsigma) W(\varsigma', \rho', \mu', y') g(\pi|a) d\pi$$

$$\text{with } \omega = (1-q)a + qz + \frac{(1-\rho)}{\rho}[(1-q)\alpha + qz] \tag{20}$$

$$\rho' = b(\pi, a, \alpha, \rho) \tag{21}$$

$$\mu' = \frac{\beta}{\beta_a(1-q)}\rho\gamma \text{ with } \mu_0 = 0 \tag{22}$$

$$y' = \frac{\beta_\alpha}{\beta_a} \frac{1}{g(\pi|a)} \left[\phi \frac{\partial g(\pi|\alpha)}{\partial \alpha} + \chi g(\pi|\alpha)\right] \text{ with } y_0 = 0 \tag{23}$$

PC slope, weight on output gap variability

- π is percent qar and x is a percent deviation.
- $\kappa = 0.08$ implies a relatively flat Phillips curve, consistent with
  - estimates from 1950s and 1960s
  - modern cost-based estimates if low marginal cost elasticity (wrt x)
- $\vartheta_x = 0.1$ translates to $(\bar{\pi} - \pi^*)^2 + 1.6\,(x - x^*)^2$ in annual inflation $\bar{\pi}$

Return to CalibTable

Discrete States and Transition

$$\Theta_t \in \{(\theta_t = 0, \tau_t = 1), (\theta_t = 0, \tau_t = 0), (\theta_t = 1, \phi_t = 1, \tau_t = 1), (\theta_t = 1, \phi_t = 1, \tau_t = 0), (\theta_t = 1, \phi_t = 0, \tau_t = 1), (\theta_t = 1, \phi_t = 0, \tau_t = 0)\}$$

with transition prob matrix $P_{i,j} = Pr(\Theta_t = j | \Theta_{t-1} = i)$:

$$\begin{bmatrix}
1-q & 0 & \delta_\rho b_{t-1}^{i=1} q & \delta_\rho (1 - b_{t-1}^{i=1}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q \\
0 & (1-q) & \delta_\rho b_{t-1}^{i=2} q & \delta_\rho (1 - b_{t-1}^{i=2}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q \\
1-q & 0 & \delta_\rho b_{t-1}^{i=3} q & \delta_\rho (1 - b_{t-1}^{i=3}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q \\
0 & (1-q) & \delta_\rho b_{t-1}^{i=4} q & \delta_\rho (1 - b_{t-1}^{i=4}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q \\
1-q & 0 & \delta_\rho b_{t-1}^{i=5} q & \delta_\rho (1 - b_{t-1}^{i=5}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q \\
0 & (1-q) & \delta_\rho b_{t-1}^{i=6} q & \delta_\rho (1 - b_{t-1}^{i=6}) q & (1-\delta_\rho)\overline{\rho} q & (1-\delta_\rho)(1-\overline{\rho}) q
\end{bmatrix}$$

where $b_{t-1}^i := b(\varsigma_{t-1}, \rho_{t-1}, \mu_{t-1}, \pi_{t-1} | \Theta_{t-1} = i)$

Return