Optimal Stake-Share Issuance Rule in Proof-of-Stake Blockchains

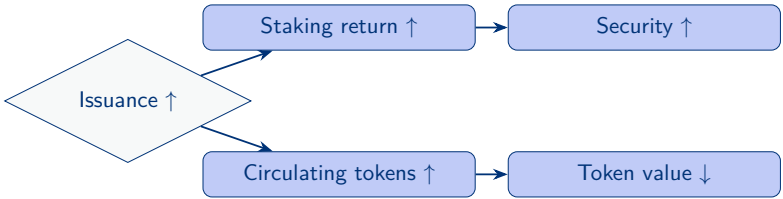
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Hong Kong Baptist University, Nov 17, 2026

Background

- Proof-of-Stake blockchains are becoming financial infrastructure.
  - Ethereum supports stablecoins, tokenized assets, smart contracts, and decentralized finance.
- ETH (native currency of Ethereum) plays two roles in this system.
  - Liquid ETH: liquid balances for transaction fees, collateral, settlement, and DeFi activity.
  - Staked ETH: locked collateral to validate transactions and earn protocol rewards.
- **Security of PoS blockchain is endogenous.**
  - Staked ETH is penalized when it is used for malicious activities.
  - It depends on the value of staked ETH backing the payment system.

Motivation

- Issuance creates a **security-valuation tradeoff**.



- Balancing the trade-off crucial for **the future role of PoS**;
  infrastructure for on-chain activities or **an investment vehicle**.
- Existing issuance rules leave a design gap.
  - Stake-level rule: responsive to staking declines, but loses long-run policy leverage.
  - Needs a new design that preserves staking responsiveness and long-run policy leverage.

What we do?

Question

How should a PoS protocol design issuance rule when it affects ledger security?

- Propose a **staked-share issuance rule** that affects long-run staking, token value, and liquidity, while still responsive to staking shocks.
- Show the long-run effects of issuance in a macro-finance model of Ethereum.
- Characterize optimal issuance rule as to deliver a targeted **optimal staked-share**.
- As transaction capacity expands, welfare may fall under fixed issuance but rises under optimal issuance.

Related Literature and Contribution

- **PoS issuance**: Jermann (2025, 2026); Jermann & Xiang (2025).

  We derive the optimal security target endogenously instead of taking a target staking ratio as given, and make it depend on capacity.

- **Blockchain economics and tokenomics**: Abadi & Brunnermeier (2018); Biais et al. (2019); Cong, Li & Wang (2022); Kiayias, Lazos & Schlegel (2025).

  We take consensus and Ethereum use as given and map PoS reward rules into staked share and ETH price growth.

- **Security and valuation**: Pagnotta (2022); John, Rivera & Saleh (2021); Sockin & Xiong (2023); Cong, He & Tang (2025); Harvey, John & Saleh (2025).

  An issuance-driven security–valuation trade-off through value at stake; the issuance rule is the policy object.

- **Optimal money issuance**: Friedman (1969); Schmitt-Grohé & Uribe (2004); Kiarsi (2018).

  PoS adds a security wedge: positive trend supply growth can be optimal.

What Is Proof-of-Stake?

- Validators replace miners as the agents who validate transactions.
- To participate, validators stake ETH as collateral.
- If a validator attacks or violates protocol rules, the stake can be penalized.
- Staking thus provides blockchain security: More staked value $\Rightarrow$ better security.
- To attract staking, the protocol rewards validators with new ETH.

Ethereum Supply

- Total ETH supply is split into staked and unstaked balances:

$$M_t = M_t^S + M_t^U.$$

- Staked ETH earns validator rewards: $i_t^m + i_t^p$

  part of rewards is newly issued ETH: i_t^m
- Unstaked ETH provides liquidity for transactions, collateral, settlement, and DeFi activity.
- Transaction is measured in "gas" unit and demands transaction fee: $f_t^b + f_t^p$

  part of fee is removed from circulation (burned): f_t^b
- Aggregate supply evolves as issuance minus fee burning:

$$M_{t+1} = M_t + i_t^m M_t^S - f_t^b G_t.$$

- Priority fee is reallocation from users to validators: $i_t^p M_t^S = f_t^p G_t$

Why the Current Rule Is Not Enough

- Ethereum's current issuance rate depends on the level of staked ETH:

$$i_t^m = \frac{k}{\sqrt{M_t^S}}.$$

- This rule is responsive: when the staking base falls, the reward rate rises.
- Even without fee burning, using $M_t^S = sM_t$,

$$\frac{M_{t+1}}{M_t} = 1 + \frac{k\sqrt{M_t^S}}{M_t} = 1 + \frac{k\sqrt{s}}{\sqrt{M_t}} \longrightarrow 1.$$

- Thus long-run supply growth is zero, independently of k.
- **The rule is robust in the short run but weak as a long-run policy instrument.**

Proposed Rule

- Replace stake-level dependence with **staked-share dependence**:

$$i_t^m = \frac{k}{s_t^{\varphi}}, \qquad s_t \equiv \frac{M_t^S}{M_t}, \qquad \varphi \in [0, 1).$$

- If the staked share falls, validator rewards rise automatically.
- Total issuance scales with M_t: k relevant for long-run supply growth and allocation.

Household's Utility

- Representative households choose consumption, gas usage, staked ETH, and liquid ETH.

$$U(C_{j,t}, G_{j,t}, m_{j,t}^U; m_t^S) = \log(C_{j,t}) + \psi \frac{\left[\left(\frac{H_{j,t}}{\eta}\right)^{\eta} \left(\frac{m_t^S}{1-\eta}\right)^{1-\eta}\right]^{1-\sigma}}{1-\sigma}.$$

- Security determined by value of total staked ETH

$$m_t^S \equiv \frac{p_t M_t^S}{Y_t}$$

- Ethereum services combine liquid ETH and gas usage:

$$H_t = \left[\alpha^{1/\xi}(m_{j,t}^U)^{(\xi-1)/\xi} + (1-\alpha)^{1/\xi}(m_t G_{j,t})^{(\xi-1)/\xi}\right]^{\xi/(\xi-1)}.$$

- ξ: whether gas usage and liquid ETH are substitutes or complements.

Interpreting the Elasticity ξ

Liquid staking tokens (LSTs, e.g., stETH) are tradable claims on staked ETH and its rewards.

- **Substitutes, $\xi > 1$:** LSTs replace liquid ETH as DeFi collateral and liquidity, so staking can earn rewards while still supporting Ethereum services.
- More on-chain activities lower marginal value of liquid ETH.
- **Complements, $\xi < 1$:** Gas payment, settlement, and margin require native liquid ETH; LSTs are imperfect substitutes because of depeg and withdrawal-delay risk.
- More on-chain activities raise marginal value of liquid ETH.

ξ determined by the extent to which LSTs can stand in for liquid ETH.

Households' Problem

$$\max_{C_{j,t},G_{j,t},M^U_{j,t+1},M^S_{j,t+1}} \sum_{s=0}^{\infty} \beta^s \, U\big(C_{j,t+s}, G_{j,t+s}, m^U_{j,t+s}; \, m^S_{t+s}\big),$$

subject to the period budget constraint:

$$C_{j,t} + p_t M^U_{j,t+1} + p_t M^S_{j,t+1} = (1 + i_t) p_t M^S_{j,t} + p_t M^U_{j,t} - p_t f_t G_{j,t} + Y_{j,t}.$$

Households take security m^S and prices (p, i, f) as given:

- **Staked ETH** is the saving asset: it earns validator rewards $i_t = i^m_t + i^p_t$.
- **Unstaked ETH** is deployable on-chain liquidity for lending, collateral, and so on.
- **Gas usage** is a transaction fee for on-chain services $f_t = f^b_t + f^p_t$.
- f^b_t is set to make $G_t = G^*$ (targeted gas usage). f^p_t proportional to f^b_t.

Balanced-Growth Path

- **Equilibrium**: A sequence of prices and quantities satisfying:
  - Household optimality: FOCs of $C_{j,t}, G_{j,t}, M^U_{j,t+1}, M^S_{j,t+1}$
  - Market clearing: $Y_t = C_t$, $M_t = M^S_t + M^U_t$.
  - Law of motion of money supply: transaction fee rule f_t + issuance rule i^m.
- **Balanced Growth Path (BGP)**: An equilibrium path that grows at various constant rates.
- If $k > -1$, unique BGP characterized by $\{s, \Pi, i^m, G^*\}$.
  - s: staked share
  - Π: Gross growth rate of ETH price p.
  - i^m: issuance rule
  - G^*: Targeted gas usage.

Optimal Rule: Overview

Household's problem vs Protocol's problem

- Security and price growth are taken as given for individual optimization.

Optimal issuance rule vs Optimal (fiat) monetary policy

- **Endogenous security**
  Higher issuance raises security, but at a price-growth cost.
- **Liquidity and security value vary with transaction capacity**
  Higher capacity may pull ETH towards DeFi use and hurt security if liquid ETH and gas usage are complements.

Protocol's Policy Objective

Define the Ethereum services composite as:

$$T(1-s;G^*) := \left[\alpha^{\frac{1}{\xi}}(1-s)^{\frac{\xi-1}{\xi}} + (1-\alpha)^{\frac{1}{\xi}}\,(G^*)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}.$$

Proposition (Policy Objectives)

Along the unique interior BGP, the representative household's blockchain-utility component is proportional to

$$W(s,\Pi;G^*) = \Pi\, s^{1-\eta}\,[T(1-s;G^*)]^{\eta}.$$

Token value growth Π, Security s, Ethereum services T.

Along BGP, Π and s move in opposite direction.

BGP relation between staked share and token value growth

Conditional on G^*, varying i^m moves s and Π along the equilibrium locus:

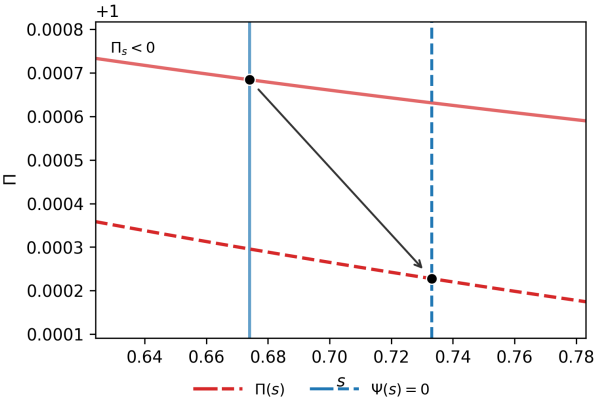
$$\Pi_{\text{BGP}}(s; G^*) \qquad (1)$$

The set of attainable staked shares s:

$$\mathcal{S}(G^*) = (s_{\min}, s_{\max}) \subset (0, 1).$$

$s_{\min}$ and $s_{\max}$ decreasing in $(G^*)^{\frac{1}{\xi}-1}$.

Higher issuance i^m: s increases, Π decreases.



Protocol's Policy Problem

Policy target: staked share s that maximizes welfare W

$$s_W^*(G^*) := \arg \sup_{s \in \mathcal{S}(G^*)} W\left(s, \Pi_{\text{BGP}}(s; G^*); G^*\right), \tag{2}$$

Proposition (Uniqueness)

If $\xi \geq \frac{1}{8}$, the welfare-maximizing staked share $s_W^*(G^*)$ is unique.

Policy instrument: issuance rate i^m

- Unique BGP: map s_W^* to i_W^{m*}
- Back out issuance parameter k by $i_W^{m*} = k/s_W^{*\varphi}$

A Growing Block-size Target

- The targeted G^* is constrained by technical and decentralization considerations.
- Greater G^* allows the network to accommodate more transactions and ease congestion.
- There is strong demand to expand Ethereum's capacity.



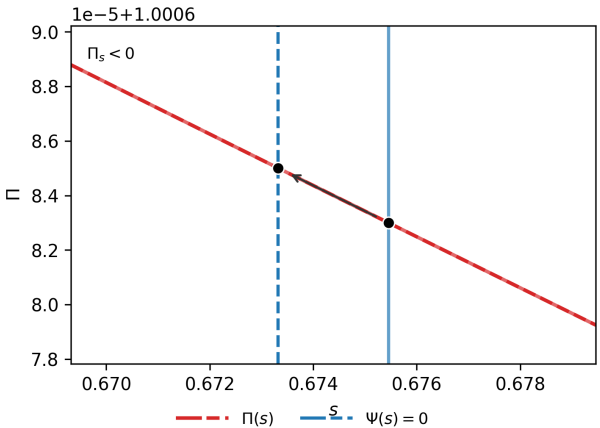
Block gas limit on Ethereum mainnet. Sources: Etherscan; Ethereum Foundation.

Effect of Growing G^* on BGP

Capacity expansion shifts ETH between staking and liquid DeFi use:

- **Substitutes, $\xi > 1$:** Liquid staking provides services without unstaking, so the staked share rises.
- **Complements, $\xi < 1$:** DeFi requires liquid ETH, so the staked share falls.

Higher issuance $(G^*)^{\frac{1}{\xi}-1}$: s decreases, Π increases.



Effect of Growing G^* on Optimal Staked Share

Proposition (Optimal Staked Share)

When $\xi > \max\{\frac{\beta}{1+\beta}, \frac{1}{8}\}$, the optimal staked share s^*

- increases with G^* if liquid ETH and gas usage are substitutes ($\xi > 1$);
- decreases with G^* if liquid ETH and gas usage are complements ($\xi < 1$);
- is independent of G^* if $\xi = 1$.

A policy that was optimal at one capacity level need not remain optimal after scaling.

- Optimal s^* differs from s_{BGP} as Π and s are taken as given in individual optimization.
- i^m adjusts to implement optimal s^*.

But when will adapting issuance optimally make a qualitative difference?

When Does Optimal Issuance Make a Qualitative Difference?

Proposition (No Welfare Divergence under Substitutes)

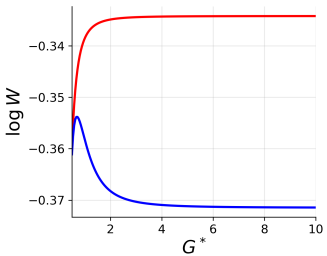
When liquid ETH and gas usage are substitutes ($\xi > 1$), increasing G^* improves welfare under both fixed and optimally adjusted issuance for sufficiently large G^*.

- Higher capacity raises staking under both issuance regimes.
- Optimal adjustment changes the magnitude, but not the direction, of the welfare effect.

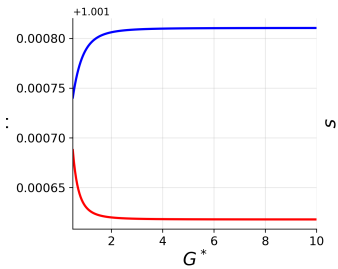
A qualitative divergence can instead arise under complementarity.

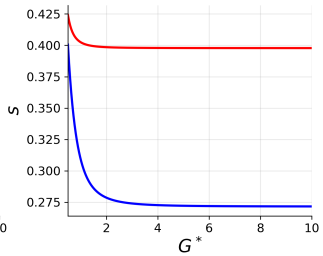
$W = \Pi\, s^{1-\eta}\, T^{\eta}$ with Fixed vs. Adjusted Issuance

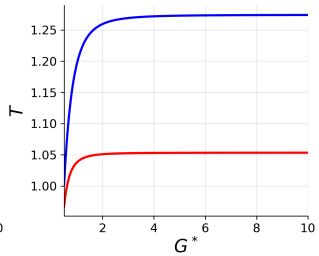
$\eta = 0.602$
$\beta = 0.999$
$\alpha = 0.665$
$\phi = 0.367$
$\zeta = 0.002$
$\xi = 0.270$
fixed $i^m = 1.100\%$ (annual)



under optimal s^*
under eq s_{BGP}







When does Welfare Implication $W = \Pi\, s^{1-\eta}\, T^{\eta}$ Diverge?

Proposition (Necessary Condition for Welfare Divergence)

When liquid ETH and gas usage are complements ($\xi < 1$), a necessary condition for welfare to decrease with G^* under fixed issuance but increase under optimally adjusted issuance for sufficiently large G^* is

$$\tilde{\eta}_W \leq \eta < \tilde{\eta}_{\mathsf{BGP}}(i^m).$$

- When $\xi < 1$, capacity expansion shifts ETH from staking to liquid use and reduces security.
- $1 - \tilde{\eta}_W$ is the upper bound on the security weight for adjusted issuance welfare to rise.
- $1 - \tilde{\eta}_{\mathsf{BGP}}(i^m)$ is the lower bound on the security weight for fixed issuance welfare to fall.

Calibration: Data

- Monthly sample: the Merge (September 2022) through June 2026.
- On-chain data from Dune: gas usage and targets, fees, consensus rewards, active stake, and the ETH/USD price.
- Total ETH supply comes from Coin Metrics; income comes from S&P Global monthly GDP.
- FRED instruments: the federal funds rate, the two-year Treasury yield, and the ten-year TIPS yield.
- We construct $s_t = M_t^S/M_t$ and $m_t = p_t M_t/Y_t$, convert daily rates to monthly rates, and normalize the last gas target to $G^* = 1$.

Calibration: Estimation Strategy

Step 1: Two-step GMM for (α, ξ)

$$\frac{i_t^b + i_t^p}{i_t} = \left[\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-s_t}{s_t}\right)\right]^{1/\xi}\left(\frac{G_t}{s_t}\right)^{1-1/\xi}.$$

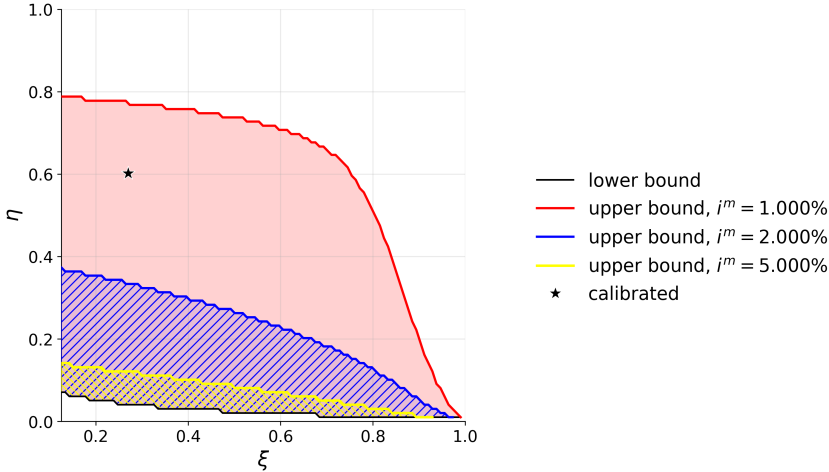
HAC Bartlett standard errors use three lags. The preferred rate-IV specification gives $\alpha = 0.665$ and $\xi = 0.270$ (Hansen $J = 0.890$).

Step 2: Linear-BGP nonlinear least squares for (η, σ, ψ)

For each month t, inherit observed $(M, s)_{t-1}$, hold G_t^* fixed, and use the first-order eq conditions to predict $(m, s, f)_t$. Choose (η, σ, ψ) to minimize weighted one-month-ahead log prediction errors, holding the GMM estimates and remaining constants fixed.

Calibration Results

Parameter	Interpretation	Value	Identification
β	Discount factor	0.9993	0.8% annual discount rate
ζ	Net endowment growth	0.00206	2.5% annual growth
α	Liquidity-services weight	0.665	GMM with rate instruments
ξ	Substitution elasticity	0.270	GMM with rate instruments
η	Ethereum-services weight	0.602	Linear-BGP NLS
σ	Blockchain-utility curvature	10.08	Linear-BGP NLS
ψ	Blockchain-utility scale	2.72×10^{-22}	Linear-BGP NLS
ϕ	Priority-fee share	0.367	Post-Merge sample mean
τ	Base-fee adjustment	0.125	EIP-1559 protocol value
G^*	Target gas usage	1	Last-sample target normalized
k	Issuance parameter	1.06×10^{-4}	BGP supply normalization



- Larger i^m: s decreases less. T dominates;
- Larger ξ: Less complementary. s decreases less.

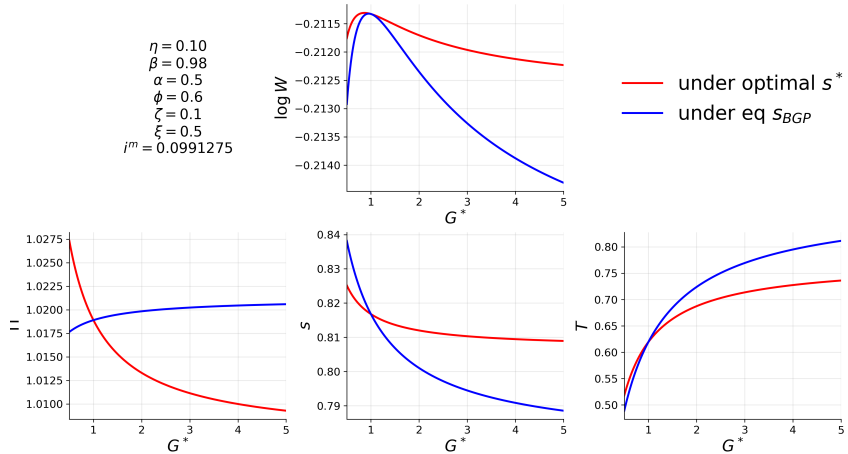
When there is an Optimal Capacity?

Denote the welfare under the optimal issuance rule as

$$W^*(G^*) := \max_s \Pi_{\text{BGP}}(s; G^*)\, s^{1-\eta}\, [T(1-s; G^*)]^\eta$$

Proposition (Optimal Capacity)

When liquid ETH and gas usage are complements ($\xi < 1$) and security is important enough $\eta < \tilde{\eta}_W$, the welfare function $W^*(G^*)$ achieves its maximum at some $G^* \in (\underline{G}, \overline{G})$. Moreover, $W^*(G^*)$ is strictly increasing in G^* for $G^* < \underline{G}$ and strictly decreasing for $G^* > \overline{G}$.



$$W = \Pi_{\text{BGP}}(s; G^*)\, s^{1-\eta}\, [T(1-s; G^*)]^{\eta}$$

- η is small, so security component dominates.

Conclusions

- **A staked-share issuance rule for PoS blockchains.**
  - Keeps rewards responsive to staking declines and restores long-run policy leverage.
- **Optimal issuance targets a staked share.**
  - Balances security, token value growth, and Ethereum services.
- **Issuance must adapt as capacity grows.**
  - Elasticity of substitution b/w G^* and liquid ETH: substitutes ($\xi > 1$) or complements ($\xi < 1$).
  - Optimal s^* rises with G^* if $\xi > 1$ and falls if $\xi < 1$.
  - If $\xi < 1$, scaling can lower welfare under fixed issuance but raise it under optimal issuance.
  - When security matters enough, there is an optimal capacity.
- **Calibration: G^* and liquid ETH are complements ($\xi = 0.27$).**